Robust LQ Control with Adaptive Law for MIMO Descriptor System

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1 Introduction

This paper presents a robust LQ control system with Model Reference Adaptive Control(MRAC) law for MIMO system which is described as descriptor form. Generally, the performance degradation is expected to happen in case that uncertainty excess the upper and lower bound which is considered in the robust control synthesis process. For this problems, adaptive control algorithms have potential to improve performance and reliability in control system. In this study, we focus on this characteristics of adaptive control algorithms and add adaptive law into usual robust control system. The proposed system is synthesized by two-step approach. First of all robust LQ controller is synthesized through solving some LMI conditions. This robust LQ controller can be handle with limited uncertainty parameters. Second, adaptive law is designed and consolidated closed loop stability of adaptive loop and robust LQ control loop is analyzed though solving quadratic stability conditions.

In this study, LMI based stability analysis method for the combined system of attached adaptive law and MIMO system described as descriptor form, whose E matrix is singular, is developed after Yang's method for SISO state space [6]. Our approach for MIMO descriptor systems is natural extension of Yang's results for state space representation. Furthermore, adaptive law introduced in this study guarantees the convergence speed. Quadratic stability is analyzed for proposed system is analyzed. Finally, the effectiveness of the proposed procedure is verified by some experiment with using 2 Degree-of-Freedom (2DOF) helicopter. Comparing the proposed method with usual robust LQ control without MRAC and nominal LQ with MARC, the effectiveness of the proposed method is validated.

2 Robust LQ controller synthesis

It is difficult to deal with uncertainties in state space representation whose dependency is not affine. In this paper, we avoid this difficulty with using descriptor representation and Linear Fractional Transformation (LFT). Consider a continuous time multi-input multi-output system described by:

$$\left(E_p + \sum_{i=1}^{k} \delta_i E_i\right) \dot{x}_p = \left(A_p + \sum_{i=1}^{k} \delta_i A_i\right) x_p + \left(B_p + \sum_{i=1}^{k} \delta_i B_i\right) u_p$$
$$y = C_p x_p \tag{1}$$

where $E_p,\ E_i,\ A_p,\ A_i \in \Re^{n\times n}$, $B_p,\ B_i \in \Re^{n\times m}$, $C_p \in \Re^{p\times n}$. Eq.(1) has affine perturbation in each coefficient matrices. Additionally, $\delta_i \in \Re$ is perturbation elements which satisfy $|\delta_i| \le 1$. For simplicity $E(\delta)$, $A(\delta)$ and $B(\delta)$ matrices are defined as :

$$E(\delta) = E_p + \sum_{i=1}^{k} \delta_i E_i, \quad A(\delta) = A_p + \sum_{i=1}^{k} \delta_i A_i, \tag{2}$$
$$B(\delta) = B_p + \sum_{i=1}^{k} \delta_i B_i.$$

Generally, it is difficult to analyze the system stability directly whose $E(\delta)$ matrix has uncertainty parameters. However, through adopting descriptor variables as $\hat{x} := [x_p^T\ \dot{x}_p^T\ u^T]$, uncertainties in each coefficient matrices are integrated into matrix A.

$$\hat{E}\dot{\hat{x}} = \hat{A}(\Delta)\hat{x} + \hat{B}u_p, \quad y = \hat{C}\hat{x} \tag{3}$$

$$\hat{A}(\Delta) = A_0 + \sum_{i=1}^{k} \delta_i A_i, \hat{E} = diag\{I,0,0,0\}, \hat{C} = [C_p\ 0\ 0]$$

$$\hat{A}(\Delta) = \begin{bmatrix} 0 & I & 0 \\ A(\delta) & -E(\delta) & B(\delta) \\ 0 & 0 & -I \end{bmatrix}, \hat{B} = \begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix}$$

Due to the uncertainties in the matrix A depend polynomially at Eq.(3), therefore Eq.(3) need to convert into equivalent model whose uncertainty terms depend as first order with adopting Linear Fractional Transformation(LFT). $\hat{A}(\Delta)$ can be represented as follows:

$$\hat{A}(\Delta) = A_0 + B_\delta \Delta C_\delta, \quad \Delta = diag[\delta_1 I_1 \cdots \delta_k I_k]. \tag{4}$$

Eq.(3) is converted into the following system with using Eq.(4).

$$\begin{aligned} \hat{E}\dot{\hat{x}} &= A_0\hat{x} + B_\delta \delta_\omega + \hat{B}u_p \\ \delta_\eta &= C_\delta \hat{x} \\ \delta_\omega &= \Delta\delta_\eta \end{aligned} \tag{5}$$

Let descriptor variable $\hat{x_d} = [\hat{x}\ \ \delta_\omega]$ then closed loop system is obtained as:

$$E_d = A_d \hat{x}_d + B_d u_p \tag{6}$$

$$\hat{E}_d = \begin{bmatrix} E_d & 0 \\ 0 & 0 \end{bmatrix}, \hat{A}_d = \begin{bmatrix} A_0 & B_\sigma \\ \Delta C_\delta & -I \end{bmatrix}, \hat{B}_d = \begin{bmatrix} \hat{B} \\ 0 \end{bmatrix}$$

Note that E_d is independent from uncertainty parameters and only A_d linearly depends on uncertainty. One integrator is added into the closed loop system. For the plant model Eq. (6), let y, r , $e_p := r - y$ and z are observable output, reference, error and integrated value of e_p, respectively. Letting state as $\tilde{x} = [z\ \ x_d^T]^T$, we finally obtain Eq.(7) for the augmented system with integrator.

$$\tilde{E_{ds}}\dot{\tilde{x}} = \tilde{A_{ds}}\tilde{x} + \tilde{B_{ds}}u_p \tag{7}$$

$$\tilde{E_{ds}} = \begin{bmatrix} I & 0 \\ 0 & E_d \end{bmatrix}, \tilde{A_{ds}} = \begin{bmatrix} 0 & -C_d \\ 0 & A_d \end{bmatrix}$$

$$\tilde{B_{ds}} = \begin{bmatrix} 0 & B_d^T \end{bmatrix}^T$$

To derive a stabilizing state feedback $u = Kx_p$, consider to minimize the following cost function.

$$J = \int_0^\infty (x_p^T Q x_p + u_p^T R u_p) dt \tag{8}$$

Where $Q \in \Re^{n\times n} > 0$ and $R \in \Re^{m\times m} > 0$ are given weighting matrices. For the redundant descriptor system, we have already obtained the following lemma in the previous research[4].

Lemma 1 If there exist $X_{11} > 0$, X_d, Y_d such that Eq. (9) hold, then the closed loop system with the state feedback $u = -Kx_p := YX_{11}^{-1}x_p$ is stable.

$$\begin{bmatrix} \mathrm{He}[A_{ds}X_d - B_dY_d] & X_d^T(Q^{\frac{1}{2}})^T & Y_d{}^T(R^{\frac{1}{2}})^T \\ Q^{\frac{1}{2}}X_d & -I & 0 \\ R^{\frac{1}{2}}Y_d & 0 & -I \end{bmatrix} < 0 \quad (9)$$

$$X_d = \begin{bmatrix} X_{11} & 0 & 0 \\ X_{21} & X_{22} & 0 \\ X_{31} & X_{32} & X_{33} \end{bmatrix}, Y_d = \begin{bmatrix} Y & 0 & 0 \end{bmatrix} \quad (10)$$

Furthermore, through maximizing the trace of X_{11}, J is guaranties $J < trace(X_{11})^{-1}$.

Synthesized controller is divided into integration gain $K_r \in \Re^{m\times m}$ and state gain $K_x \in \Re^{m\times n}$ as $K = [K_r \ \ K_x]$. Where u_{nom} is nominal input using robust LQ state feedback for reference and actual model and $r(t)$ is step reference.

$$u_{nom} = -K_x x_p + K_r \int (r(t) - y)dt \quad (11)$$

3 Adaptive law with σ modification for descriptor system

In this section, adaptive law and quadratic stability analysis for adaptive control loop are discussed. Yang et al [6] developed a LMI-based stability analysis method that employs σ-modification for SISO system. In this study, we expand Yang et al's method to MIMO descriptor system.

Consider the the MIMO system described as descriptor system. For Eq.(1), let descriptor variable is $\hat{x}_p = [x_p \ \ \dot{x}_p]^T$ then Eq.(1) is described as follows.

$$\hat{E}_p\dot{\hat{x}}_p = \hat{A}_p\hat{x}_p + \hat{B}_p(u + W^T\phi(\hat{x}_p)), \ y = \hat{C}_p\hat{x}_p \quad (12)$$

$$\hat{E}_p = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, \hat{A}_p = \begin{bmatrix} 0 & I \\ A(\delta) & -E(\delta) \end{bmatrix},$$
$$\hat{B}_p = \begin{bmatrix} 0 & B(\delta)^T \end{bmatrix}^T, \hat{C}_p = \begin{bmatrix} C_p & 0 \end{bmatrix}$$

Where $W(t) = [W_1(t)\cdots W_m(t)] \in \Re^{2n\times m}$, $W_i(t) \in \Re^{2n\times 1}$ is uncertain parameter matrix and $\phi(\hat{x}_p) \in \Re^{2n\times 1}$ is a known set of smooth basis functions. $W^T(t)\phi(\hat{x}_p)$is a matched system uncertainty. Actual input u for the argued system is describe as follows.

$$u = u_{nom} - u_{ad}, \ \ u_{ad} = \hat{W}(t)^T\phi(\hat{x}_p) \quad (13)$$

Where u_{nom} is the nominal input for reference model derived in the previous section and u_{ad} is the adaptive signal. u_{ad} functions as canceling matched uncertainty $\hat{W}^T\phi(\hat{x}_p)$ through estimating the uncertain parameter matrix $W(t)$ with $\hat{W}(t) = [\hat{W}_1(t)\cdots\hat{W}_m(t)] \in \Re^{2n\times m}$, $\hat{W}_i(t) \in \Re^{2n\times 1}$. Reference model which generates ideal output for Eq.(12) is described as follows:

$$\text{Reference model}: \quad \hat{E}_p\dot{\hat{x}}_m = \hat{A}_m\hat{x}_m + \hat{B}_m\int(r(t) - y)dt \quad (14)$$

Where $\hat{A}_m$ and $\hat{B}_m$ as $\hat{A}_m = \hat{A}_p - \hat{B}_p[K_x \ \ 0], \hat{B}_m = \hat{B}_pK_r$. Let $e = \hat{x}_m - \hat{x}_p$ is tracking error and $\tilde{W}(t) = \hat{W}(t) - W(t)$ $(\tilde{W}_1(t) = \hat{W}_1(t) - W_1(t)\cdots\tilde{W}_m(t) = \hat{W}_m(t) - W_m(t))$ is the estimation error. Let $\hat{B}_p = [\hat{B}_{p1}\cdots\hat{B}_{pm}] \in \Re^{2n\times m}$, $\hat{B}_{pi} \in \Re^{2n\times 1}$. Finally the error between Eq.(12) and Eq.(14) is obtained as Eq.(15).

$$\hat{E}_p\dot{e} = \hat{A}_me + \hat{B}_p\tilde{W}(t)^T\phi(\hat{x}_p) \quad (15)$$

$\hat{W}(t)$ are updated using Eq. (16) as adaptive law with σ-modification[6], [7], [8].

$$\dot{\hat{W}}(t) = -\gamma\phi(\hat{x}_p)e^T\hat{P}\hat{B}_p - \sigma\hat{W}(t) \quad (16)$$

Where $\gamma > 0 \in \Re$ is adaptive gain and σ is σ-modification gain. The matrix $\hat{P} > 0$ in Eq.(16) satisfies following LMI condition Eq.(17).

Lemma 2 If there exists $\hat{P} > 0$ such that Eq.(17) hold, then state vector $\hat{x}_p$ in Eq.(12) is exponentially bounded by:
$\|\hat{x}_p(t)\| < \sqrt{\frac{\lambda_{max}(\hat{P})}{\lambda_{min}(\hat{P})}}e^{-\rho t}\|\hat{x}_p(0)\|$.

$$\hat{A}_m^T\hat{P}^T + \hat{P}\hat{A}_m + 2\rho\hat{E}_p\hat{P} < 0, \ \ \hat{P} = \begin{bmatrix} P_{11} & 0 \\ P_{12}^T & P_{22} \end{bmatrix} \quad (17)$$

Proof: Considering following Lyapnov function $V(\hat{x}_p)$,

$$V(\hat{x}_p) = \hat{x}_p^T\hat{E}_p\hat{P}\hat{x}_p \quad (18)$$
$$\dot{V}(\hat{x}_p) = \hat{x}_p^T(\hat{A}_m^T\hat{P}^T + \hat{P}\hat{A}_m)\hat{x}_p < \hat{x}_p^T(-2\rho\hat{E}_p\hat{P})\hat{x}_p = -2\rho V(\hat{x}_p)$$

$V(\hat{x}_p)$ satisfies $V(\hat{x}_p) < e^{-2\rho t}V(\hat{x}_p(0))$. Furthermore through adopting $\lambda_{min}(\hat{P})\|\hat{x}_p\|^2 \le \hat{x}_p^T\hat{P}\hat{x}_p \le \lambda_{max}(\hat{P})\|\hat{x}_p\|^2$, we obtained following relationship:

$$\lambda_{min}(\hat{P})\|\hat{x}_p(t)\|^2 \le \hat{x}_p^T\hat{P}\hat{x}_p < e^{-2\rho t}\hat{x}_p^T(0)\hat{P}\hat{x}_p(0) \quad (19)$$
$$\le e^{-2\rho t}\lambda_{max}(\hat{P})\|\hat{x}_p(0)\|^2.$$

From Eq.(19) state vector $\hat{x}_p(t)$ converge to 0 with faster than convergence rate ρ.

Letting $\zeta = [\tilde{W}_1^T\cdots\tilde{W}_m^T \ \ e]^T$ as error dynamics variables, the consolidated error dynamics whose descriptor variable is consist of the tracking error and weight estimation error is described as Eq.(20).

$$\check{E}\dot{\zeta} = \check{A}\zeta + \check{B}\sigma W \quad (20)$$

$$\check{A} = \begin{bmatrix} -\sigma I_N & 0 & 0 & -\gamma\phi(\hat{x})B_1^T\hat{P} \\ 0 & \ddots & 0 & \vdots \\ 0 & 0 & -\sigma I_N & -\gamma\phi(\hat{x})B_n^T\hat{P} \\ B_1\phi(\bar{x}) & \cdots & B_n\phi(\hat{x}) & A_m \end{bmatrix},$$
$$\check{B} = \begin{bmatrix} -I_{N\times n} \\ 0 \end{bmatrix}, \check{E} = \begin{bmatrix} I & 0 \\ 0 & E_p \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}$$

A stability analysis for the system Eq.(20) is carried out by considering the following Lyapnov function.

$$V(\zeta) = e^T\hat{E}_p\hat{P}e + \frac{1}{\gamma}\tilde{W}(t)^T\tilde{W}(t)^T \quad (21)$$

The time derivative of Lyapnov function Eq.(21) $\dot{V}(\zeta)$ is calculated as follows.

$$\dot{V}(\zeta) = -2\rho e^T\hat{E}_p\hat{P}e - 2\frac{\sigma}{\gamma}\tilde{W}^T\hat{W} \quad (22)$$
$$= -2\rho e^T\hat{E}_p\hat{P}e - \frac{\sigma}{\gamma}[\|\tilde{W}\|^2 + \|\hat{W}\|^2 - \|W\|^2]$$
$$\le -\left[-2\rho\lambda_{min}(\hat{E}_p\hat{P}) \ \ \frac{\sigma}{\gamma}\right]\|\zeta\|^2 + \frac{\sigma}{\gamma}\|W\|^2 \quad (23)$$

Eq.(23) is not negative semidefinite for small values of e, this means σ-modification increase robustness at the expense of the precise convergence of e to the origin.

Where $q = \phi(\hat{x}) = [\phi_1(\hat{x}), \ldots, \phi_N(\hat{x})]^T$is a set of basis functions. Each vertex of the uncertainty region is defined as: $q_i \in [\underline{q}_i, \overline{q}_i]$ Eq.(20) is considered as LPV system with respect to q. For the uncertainties, let $\check{A}$ as Eq. (24).

$$\check{A} = \sum_{i=1}^{n} a_i \check{A}_i, \ \sum_{i=1}^{n} a_i = 1, \ a_i \geq 0 \tag{24}$$

The following lemma is already obtained for stability analysis of descriptor systems for Eq.(20) [6]. Quadratic stability is analyzed by solving Eq.(25) at each vertexes of $\phi(\cdot)$.

Lemma 3 Eq.(20) is quadratically stable for perturbation ϕ_j if there exists $X_{11} > 0$ such that

$$\hat{X}^T \check{A}_n^T + \check{A}_n \hat{X} < 0, \qquad n = 1, \ldots 2^n \tag{25}$$

$$\hat{X} = \begin{bmatrix} X_{11} & 0 \\ X_{21} & X_{22} \end{bmatrix}$$

hold.

Where σ and γ are analyzed whether satisfy the quadratic stability through solving above LMI at each vertices.

4 Illustrative example

Proposed system is experimented with using test scale 2 DOF(Degree-Of-Freedom) helicopter to illustrate the main results of this paper. Let input $u_p(t)$ as $u_p(t) = [V_{m,p}\ V_{m,y}]^T$. $V_{m,p}$ and $V_{m,y}$ are input voltage to pitch motor and yaw motor respectively. The pitch angle θ and the yaw angle ψ are available as measurement outputs. The robustness of the proposed system is validated through adding weight below the center of pitch propeller. Therefore controlled plant is described as the model which depends on m_{add}(mass of the added weight) as not simply affine. m_{add} is considered as the uncertainty parameter in deriving robust controller. The uncertainty is assumed that true value of m_{add} exists in between 0 and 30[g]. Let $\hat{x} := [\theta\ \psi\ \dot{\theta}\ \dot{\psi}\ \ddot{\theta}\ddot{\psi}\ u_p^T]^T$ as descriptor variable, then plant dynamics is described as Eq.(26). Due to the limitation of the space, the detail of derivation of Eq.(26) and breakdown of coefficient matrices are omitted. Details are in our full paper.

$$\hat{E}_p \dot{\hat{x}} = \hat{A}_p \hat{x} + \hat{B}_p u_p \tag{26}$$

There exist squared terms of uncertainty parameter m_{add} in matrix $\hat{A}_p$. LFT (Linear Fractional Transformation) is adopted to obtain an equivalent model without squared term. Let $\hat{A}_p$ as $\hat{A}_p =: \hat{A}_0 + B_{\delta 1} m_{add} C_{\delta 1} + B_{\delta 2} m_{add} m_{add} C_{\delta 2}$ and LFT is applied into argued system [4], Eq. (27) is obtained.

$$\begin{aligned} &\hat{E}_p \dot{\hat{x}} = \hat{A}_0 \hat{x} + B_{\delta 1} w_1 + B_{\delta 2} w_2 + \hat{B}_p u_p \\ &w_1 = m_{add} C_{\delta 1} \hat{x} \\ &w_2 = m_{add} C_{\delta 2} \hat{x} \end{aligned} \tag{27}$$

By letting $x_d := [\hat{x}^T\ \ w_1\ \ w_2]^T$ as descriptor variables, Eq. (27) is finally described as Eq. (28).

$$E_d \dot{x}_d = A_d x_d + B_d u_p \tag{28}$$

$$A_d = \begin{bmatrix} \tilde{A}_0 & B_{\delta 1} & B_{\delta 2} m_{add} \\ m_{add} C_{\delta 1} & -I & 0 \\ m_{add} C_{\delta 2} & 0 & -I \end{bmatrix} \tag{29}$$

$$E_d = \begin{bmatrix} \hat{E}_p & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, B_d = \begin{bmatrix} \hat{B}_p^T & 0 & 0 \end{bmatrix}^T$$

Note that E_d is independent of the uncertain parameter m_{add} and A_d depends linearly on m_{add}. let y, r , $e_p := r - y$ and z are observable output, reference, error and integrated value of e_p, respectively. Letting state as $\tilde{x} = [z\ \ x_d^T]^T$, we finally obtain Eq.(30) for the augmented system with integrator.

$$\tilde{E}_{ds} \dot{\tilde{x}} = \tilde{A}_{ds} \tilde{x} + \tilde{B}_{ds} u_p \tag{30}$$

$$\tilde{E}_{ds} = \begin{bmatrix} I & 0 \\ 0 & E_d \end{bmatrix}, \tilde{A}_{ds} = \begin{bmatrix} 0 & -C_d \\ 0 & A_d \end{bmatrix}$$

$$\tilde{B}_{ds} = \begin{bmatrix} 0 & B_d^T \end{bmatrix}^T$$

Cost function J shown in section 2.3 is considered to derive a stabilizing state feedback $u = K\hat{x}$. Through maximizing the trace of X_{11}, J is guaranteed as $J < trace(X_{11})^{-1}$. For the uncertainties $0 \leq m_{add} \leq 30$, let A_{ds} as Eq. (31), and we expect that the true plant stays in the polytope (31).

$$A_{ds} = (1 - \alpha) A_{ds0} + \alpha A_{ds1}, 0 \leq \alpha \leq 1 \tag{31}$$

A_{ds0} and A_{ds1} are the vertices of the polytope of the uncertain parameters, i.e. A_{ds0} is A_{ds} with m_{add}=0 and A_{ds1} is A_{ds} with m_{add}=30, respectively.

We derive a single state feedback law $u = K\hat{x}$ that minimize cost function J for the system whose coefficient matrices described by polytope (31).

$$\begin{bmatrix} \mathrm{He}[A_{dsi} X_d - B_{ds} Y_d] & X_d^T (Q^{\frac{1}{2}})^T & Y_d{}^T (R^{\frac{1}{2}})^T \\ Q^{\frac{1}{2}} X_d & -I & 0 \\ R^{\frac{1}{2}} Y_d & 0 & -I \end{bmatrix} < 0 \tag{32}$$

$$i = 0, 1$$

$$\text{maximize}: \quad \text{trace}(X_{11}) \tag{33}$$

For the weighting matrices Q=diag$[80, 90, 150, 150, 100, 200]$, R=diag $[0.5, 0.5]$, finally the following state feedback gain K is obtained. Controller gain K is divided into integration gain $K_r \in \Re^{2\times 2}$ and state feedback gain $K_x \in \Re^{2\times 4}$. From here adaptive law is designed after previous section. Let descriptor variables of our controlled plant Eq.(26) as $\hat{x}_p := [\theta,\ \psi,\ \dot{\theta},\ \dot{\psi},\ \ddot{\theta},\ \ddot{\psi}]^T$. Then actual plant is described as Eq.(12). Detail of each coefficient matrices are shown in our full paper.

In this study weight is added below the pitch propeller to verify the effectiveness of proposed method. Hence, coefficient matrix $\hat{A}_p$ depends only polynomially on the uncertain parameters. Actually it is possible to obtain usual state space model for our plant, however there exist rational terms of uncertain parameter in the matrices A and B. For this reason, redundant descriptor representation is appropriate to describe the actual plant and the reference model. An LMI-based stability analysis method for SISO system, which is described as usual state space, is discussed in [6]. In this study LMI based stability analysis method for the MIMO system, which is described as descriptor form, is developed after Yang's method. Uncertain parameter matrix is $W(t) = [W_1(t)\ W_2(t)]$ and estimated by $\hat{W} = [\hat{W}_1(t)\ \hat{W}_2(t)]$ furthermore, known set of smooth basis function is $\phi(\hat{x}_p) \in \Re^{6\times 1}$. Adaptive law with σ-modification is described as Eq.(34).

$$\begin{aligned} \dot{\hat{W}}_1 &= -\gamma\phi(\hat{x}_p)e^T\hat{P}\hat{B}_1 - \sigma\hat{W}_1 \\ \dot{\hat{W}}_2 &= -\gamma\phi(\hat{x}_p)e^T\hat{P}\hat{B}_2 - \sigma\hat{W}_2 \end{aligned} \tag{34}$$

Through solving Eq.(17) at the convergence rate $\rho = 1$, matrix $\hat{P}$ for Eq.(34) is obtained. Let $\hat{B}_1$, $\hat{B}_2$ as $[\hat{B}_1\ \hat{B}_2] = \hat{B}_p \in \Re^{6\times 2}$ and estimation error $\tilde{W}(t) = \hat{W}(t) - W(t) := (\tilde{W}_1(t) = \hat{W}_1(t) - W_1(t),\ \tilde{W}_2(t) = \hat{W}_2(t) - W_2(t))$. The error dynamics is obtained as follows:

$$\hat{E}_p\dot{e} = \hat{A}_m e + \hat{B}_1\tilde{W}_1(t)^T\phi(\hat{x}_p) + \hat{B}_2\tilde{W}_2(t)^T\phi(\hat{x}_p) \tag{35}$$

Considering of adaptive law Eq.(34), let $\zeta = [\tilde{W}_1^T\ \tilde{W}_2^T\ e]^T$ as error dynamics variables, consolidated error dynamics is obtained as follows.

$$\hat{E}_d\dot{\zeta} = \breve{A}\zeta + \breve{B}\sigma W \tag{36}$$

$$\breve{A} = \begin{bmatrix} -\sigma I_N & 0 & -\gamma\phi(\hat{x}_p)\hat{B}_1^T\hat{P} \\ 0 & -\sigma I_N & -\gamma\phi(\hat{x}_p)\hat{B}_2^T\hat{P} \\ \hat{B}_1\phi(\hat{x}_p)^T & \hat{B}_2\phi(\hat{x}_p)^T & \hat{A}_m \end{bmatrix},$$
$$\breve{B} = \begin{bmatrix} -I_N \\ 0 \end{bmatrix}, \hat{E}_d = \begin{bmatrix} I & 0 \\ 0 & E_p \end{bmatrix}$$

Eq.(36) is considered as LPV system with respect to $\hat{x}_p = [\theta, \psi, \dot{\theta}, \dot{\psi}, \ddot{\theta}, \ddot{\psi}]^T$. Uncertain parameters exist in only 5th and 6th rows in the coefficient matrix $\hat{A}_p$ and $\hat{B}_p$. 5th and 6th descriptor variables represents $\ddot{\theta}, \ddot{\psi}$ respectively. Furthermore, there is no uncertain parameters in 1st-4th rows in $\hat{A}_p$ and $\hat{B}_p$. Hence, region of basis function is decided through considering $\ddot{\theta}$ and $\ddot{\psi}$. Let basis function $\phi(x) = [0, 0, 0, 0, \phi_5(\ddot{\theta}), \phi_6(\ddot{\psi})]^T = [0, 0, 0, 0, \ddot{\theta}, \ddot{\psi}]$ and assumed $\phi_5(\ddot{\theta}) \in [-11.6487, 11.5529]$, $\phi_6(\ddot{\psi}) \in [-5.1646, 5.1590]$ from some experimentations. For the uncertainties, let $\breve{A}$ as Eq. (37).

$$\breve{A} = \sum_{i=1}^{4} a_i\breve{A}_i,\ \sum_{i=1}^{4} a_i = 1,\ a_i \geq 0 \tag{37}$$

Quadratic stability is analyzed by solving following Eq.(38) at each vertexes of $\phi(\cdot)$.

$$\begin{aligned} &\hat{X}^T\breve{A}_n^T + \breve{A}_n\hat{X} < 0, \qquad n = 1, 2, 3, 4 \\ &\hat{X} = \begin{bmatrix} X_{11} & 0 \\ X_{21} & X_{22} \end{bmatrix} \end{aligned} \tag{38}$$

Adaptive gains γ and σ which satisfy above LMI is obtained as $\gamma = 10.5, \sigma = 0.3$. These values are used in the experiments.

5 Experiments

In this experiment, we add a weight of 45.5[g] at the helicopter to verify the robust control performance. This weight $m_{add} = 45.5$ is larger than considered range $(0 \leq m_{add} \leq 30)$. Therefore usual robust LQ control is expected to occur performance degradation. The robustness of the proposed method (Robust LQ with MRAC) is verified by comparison with robust LQ without MRAC and nominal LQ with MRAC.

For more clearly comparison, the norm of difference between ideal model output and actual output is computed as: $\frac{1}{T}\int_0^T \|e(t)\|dt$, $T = 90$. Pitch control is computed and results of experiment 2 are as follows:

- Robust LQ (Pitch): 2.894[deg]
- Nominal LQ with MRAC law(Pitch): 2.701[deg]
- Robust LQ with MRAC law(Pitch): 2.536[deg]

Yaw control is computed as follows:

- Robust LQ (Yaw): 0.58999[deg]
- Nominal LQ with MRAC law(Yaw): 0.6003[deg]
- Robust LQ with MRAC law(Yaw): 0.4976[deg]

Proposed method reduces the norms between actual and ideal output in case that parameter perturbation excess the considered upper and lower bound both pitch and yaw control. This result shows the effectiveness of the combination of robust controller and MRAC law.

6 Conclusion

In this study, we designed robust LQ control system with MRAC law. For added adaptive control loop, LMI based stability analysis method for MIMO descriptor system is developed based on former research[6]. The effectiveness of proposed system is verified by the experiments. Output of proposed system is closer to ideal model's output than that of usual robust LQ control without MRAC and nominal LQ with MRAC. From these experimental results, it can be said that the proposed method, that is the combination of robust controller and MRAC law, is able to improve the robust control performance.

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