Gain-Scheduling Control for Crane via Parameter Dependent Lyapunov Function Considering All Varying Parameters in Dynamics

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1 Introduction

In this paper, Gain-Scheduling (GS) control via parameter dependent Lyapunov function for crane is synthesized. A purpose of cranes system is to lift and carry a load to assigned positions. Cranes are required to transport the load fast, accurately and to reduce oscillations. In previous research, a method to robust control of cranes to suppress effects by variation of the rope length have been reported[1]. For improvement of cranes efficiency, cranes should be operated in movement of the load horizontal and vertical directions simultaneously. It has been reported that decentralized control is effective for cranes[2]. We have reported a method to synthesis GS control via parameter dependent Lyapunov function in concept of treating complete dynamics[3]. The crane model not to ignore velocity and acceleration of the rope length in the dynamics has been derived. However, a mass of the load is not constant and included in the dynamics. Variation of the mass of the load is not considered. By using parameter dependent Lyapunov function, a controller becomes less conservative synthesis in comparison with adopting parameter independent Lyapunov function[4]. However, there is Lyapunov matrix derivative. For this problem, a estimation method of Lyapunov matrix derivative has been reported[5].

In this paper, a GS controller via parameter dependent Lyapunov function which guarantees variation of the rope length, its velocity, acceleration and the mass of the load is synthesized. All varying parameters in the dynamics which is related to operating the crane are taken into account. The rope length, its velocity and acceleration are treated as time-varying parameters. The mass of the load is different in every operation, it is treated as uncertain but time-invariant parameter. Time-varying parameters are assumed to measurable, they are treated as scheduling parameters. Linear Parameter Varying (LPV) system for scheduling parameters is obtained. Robust stability with vertexes of varying parameters is guaranteed. By using parameter dependent Lyapunov function, there are product of scheduling parameters and derivative of Lyapunov matrix. For these problem, descriptor representation, Linear Fractional Transformation (LFT) and a estimation method of Lyapunov matrix derivative are adopted. The former problem of parameter dependent Lyapunov function can be solved by adopting descriptor representation. We show that adopting descriptor representation and LFT, the controller performance is improved from a viewpoint of evaluated function. By several simulations, it is shown that the controller is effective.

2 Controlled plant

A schematic diagram of the crane is shown in Fig. 1. A position of the trolley ξ [m], a swing angle of the load ϕ [rad] and the rope length l_p [m] are measurable. In addition, it is assumed that velocity and acceleration of the rope length can be obtained. The controlled output is the horizontal position of the load y [m]($y = \xi - l\sin\phi$). The control input is a current of a jib motor I_j [A]. The mass of the load is m_p. The mass of the

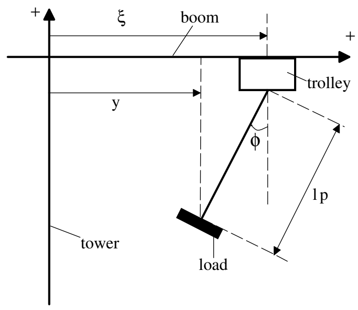

Figure 1 Crane model

trolley is m_j, the jib motor equivalent moment of inertia is J_ψ, the ratio of the jib motor gear is $K_{g,j}$, the radius of the jib motor gear is $r_{j,p}$, the torque constant of the jib motor is K_t and the gravitational acceleration is g. In this paper, the followings are assumed. i) The rope is rigid rod without the mass. ii) The load is a material point. iii) The load moves in a plane consist of the tower and the boom. iv) The friction between the trolley and the boom is ignored. Let generalized coordinate be $q(t) = [\xi(t) \quad \phi(t)]^{\mathrm{T}}$. ϕ and its velocity are assumed as small enough. $\sin\phi \simeq \phi$, $\cos\phi \simeq 1$, $\dot{\phi}^2 \simeq 0$. Let m_j be $m_t + \frac{J_\psi K_{g,j}^2}{r_{j,p}^2}$. The mathematical model is obtained as Eq.(1)

$$E\ddot{q} + F\dot{q} + Gq = HI_j \tag{1}$$

$$E = \begin{bmatrix} m_p + m_j & -m_p l_p \\ 1 & -l_p \end{bmatrix},\ F = \begin{bmatrix} 0 & -2m_p\dot{l}_p \\ 0 & -2\dot{l}_p \end{bmatrix}$$
$$G = \begin{bmatrix} 0 & -m_p\ddot{l}_p \\ 0 & -g \end{bmatrix},\ H = \begin{bmatrix} k_t \\ 0 \end{bmatrix}$$

Let state variable be $x_j = \begin{bmatrix} q^{\mathrm{T}} & \dot{q}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}$, output be y and input be $u = I_j$. Then, the state space representation of the crane is obtained as Eq.(2) and Eq.(3).

$$\dot{x}_j = A_j x_j + B_j u \tag{2}$$
$$y = C_j x_j \tag{3}$$

$$A_j = \begin{bmatrix} I & 0 \\ 0 & E \end{bmatrix}^{-1} \begin{bmatrix} 0 & I \\ -G & -F \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{m_p(\ddot{l}_p - g)}{m_j} & 0 & 0 \\ 0 & \frac{m_p\ddot{l}_p - (m_p+m_j)g}{m_j l_p} & 0 & \frac{2\dot{l}_p}{l_p} \end{bmatrix}$$

$$B_j = \begin{bmatrix} I & 0 \\ 0 & E \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ H \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \frac{K_t}{m_j} \\ \frac{K_t}{m_j l_p} \end{bmatrix}$$

$$C_j = [1 \quad -l_p \quad 0 \quad 0]$$

3 Controller synthesis

Scheduling parameters are not only the rope length l_p, but also the velocity $\dot{l}_p$ and the acceleration $\ddot{l}_p$. The stability of the closed system should be guaranteed theoretically for variation of the rope length, its the velocity and acceleration. Operation time can be shorten because it is possible to move the trolley and to hoist the rope at same time. An important purpose of controlling crane system is that the load follows references without error. The integral of the error x_e is added to the state variable x_j to make the controlled output y follows to a reference r. The augmented system and state variable $x(t)$ are assigned as Eq.(4), (5).

$$x(t) = \begin{bmatrix} x_e(t)^{\mathrm{T}} & q(t)^{\mathrm{T}} & \dot{q}(t)^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \tag{4}$$

$$x_e = \int_0^t e(\tau)\mathrm{d}\tau, e = r - y$$

$$\dot{x} = A_e x + B_e u \tag{5}$$

$$A_e = \begin{bmatrix} 0 & -Cp \\ 0 & A_p \end{bmatrix}, \ B_e = \begin{bmatrix} 0 \\ B_p \end{bmatrix}$$

3.1 Matrices Transformation

Stability conditions with state space representation are obtained as Eq.(6), (7).

$$X(\theta) = X(\theta)^{\mathrm{T}} > 0 \tag{6}$$

$$\mathrm{He}\{A_e(\theta)X(\theta) + B_e(\theta)Y(\theta)\} - \dot{X}(\theta) < 0 \tag{7}$$

Where, state feedback controller $u = K(\theta)x$ and feedback gain $K(\theta) = Y(\theta)X(\theta)^{-1}$. The coefficient matrices A_e and B_e include scheduling parameters. When GS control system is synthesized with a framework of the state space representation of the crane, there are products of scheduling parameters in the coefficient matrices and variable matrix $Y(\theta)$, scheduling parameters in the coefficient matrices and Lyapunov matrix $X(\theta)$. Linear Matrix Inequality (LMI) conditions for deriving GS controller can not be solved. Furthermore, the matrix A_e includes l and $\frac{1}{l}$. If a method in framework of polytopic is adopted for Eq.(5), then synthesized controller becomes conservative. In this paper, descriptor representation is adopted. Coefficient matrix A_e which includes an affine system for $l, \dot{l}, \ddot{l}$ and coefficient matrix B_e which does not include scheduling parameters are obtained. The parameter box Eq.(8) is defined by vertexes which consist of upper and lower bounds of the varying parameters. The scheduling parameters are $\theta = [l_p, \dot{l}_p, \ddot{l}_p]$ and its derivatives are $\dot{\theta}$. The time-invariant parameter is m_p.

$$\Theta = \{[\eta \mid \theta] = [\theta_0 \mid \theta_1,\ \theta_2,\ \theta_3,\ \theta_4] : \ \theta_i \in \{\underline{\theta}_i,\ \overline{\theta}_i\}\} \tag{8}$$

$$\theta_0 = m_p,\ \theta_1 = l,\ \theta_2 = \dot{l},\ \theta_3 = \ddot{l},\ \theta_4 = \dddot{l},\ (i = 0, ..., 4)$$

Let descriptor variable be $x_d = [x^{\mathrm{T}} \quad \ddot{q}^{\mathrm{T}}]^{\mathrm{T}}$ and matrix J be $[1 \quad -\theta_1]$. Eq.(9) is obtained from Eq.(2).

$$E_d \dot{x}_d = A_d(\theta)x_d + B_d u \tag{9}$$

$$E_d = \text{block diag}(I,\ I,\ 1,\ 0)$$

$$A_d = \begin{bmatrix} 0 & -J & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \\ 0 & -G & -F & -E \end{bmatrix}, \ B_d = \begin{bmatrix} 0 \\ 0 \\ 0 \\ H \end{bmatrix}$$

Where, I is the unit matrix which has the appropriate dimension. From Eq.(9), matrix H includes only constant. The coefficient matrix A_d includes only affine terms. If Eq.(9) includes scheduling parameters between first row and fifth row, then flexibility of the synthesized controller is fall. LFT is adopted into Eq.(9) for remove scheduling parameters between first row and fifth row. Let new descriptor variable be $x_{lf} = [x_d^{\mathrm{T}} \quad z_\delta^{\mathrm{T}}]^{\mathrm{T}}$.

$$E_{lf}\dot{x}_{lf} = A_{lf}(\theta)x_{lf} + B_{lf}u \tag{10}$$

$$A_{lf}(\theta) = A_{lf0} + \sum_{i=1}^{3} \theta_i A_{lfi} \tag{11}$$

$$= \left[\begin{array}{c|c} A_n & B_\delta \Delta \\ \hline C_\delta & -I + D_\delta \Delta \end{array}\right]$$

$$= \left[\begin{array}{c|c|c} A_{n11} & A_{n12} & B_{\delta 1}\Delta \\ \hline A_{n21} & A_{n22} & B_{\delta 2}\Delta \\ \hline C_{\delta 1} & C_{\delta 2} & -I + D_\delta \Delta \end{array}\right]$$

$$E_{lf} = \left[\begin{array}{c|c} E_d & 0 \\ \hline 0 & 0 \end{array}\right], \ B_{lf} = \begin{bmatrix} B_d \\ 0 \end{bmatrix}$$

$$A_n = \left[\begin{array}{ccccc|cc} 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \hline 0 & 0 & 0 & 0 & 0 & -(\theta_0 + m_j) & \theta_0\theta_1 \\ 0 & 0 & g & 0 & 0 & -1 & \theta_1 \end{array}\right]$$

$$B_\delta = \left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \hline 0 & \theta_0 & \theta_0 \\ 0 & 1 & 0 \end{array}\right]$$

$$C_\delta = \left[\begin{array}{ccccc|cc} 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$D_\delta = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \Delta = \mathrm{diag}(\theta_1,\ \theta_2,\ \theta_3)$$

Where, z_δ is virtual input. From Eq.(10), the coefficient matrix A_{lf} includes only affine terms and does not include scheduling parameters between first row and fifth row.

3.2 Stability conditions

The stability conditions of descriptor system which is represented by Eq.(10) is related to LQ control specification for the closed loop system assigned by state feedback controller $u = K_{lf}(\theta)x_{lf}$. Considering structure of the matrix E_{lf}, candidates of Lyapunov matrix $X_{lf}(\theta)$

and variable matrix $Y_{lf}(\theta)$ are restricted as Eq.(12).

$$X_{lf}(\theta) = \begin{bmatrix} X(\theta) & 0 & 0 \\ X_{2.1}(\theta) & X_{2.2}(\theta) & X_{2.3}(\theta) \\ X_{3.1}(\theta) & X_{3.2}(\theta) & X_{3.3}(\theta) \end{bmatrix} \tag{12}$$

$$E_{lf}\dot{X}_l(\theta) = \begin{bmatrix} \dot{X}(\theta) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ Y_{lf}(\theta) = [Y(\theta) \quad 0 \quad 0]$$

[Theorem][6] The closed loop system with a plant Eq.(9) and state feedback is stable, if $X(\theta)$, $X_{lf}(\theta)$ and $Y_{lf}(\theta)$ exist satisfying Eq.(13) and (14).

$$X(\theta) > 0 \tag{13}$$

$$\mathrm{He}\{A_{lf}(\theta)X_{lf}(\theta) + B_{lf}Y_{lf}(\theta)\} - E_{lf}\dot{X}_{lf}(\theta) < 0 \tag{14}$$

Eq.(15) is obtained when $[I \quad B_\delta\Delta(I - D_\delta\Delta)^{-1}]$ and its transportation are multiplied to Eq.(14) from both sides.

$$\mathrm{He}\left\{A_e(\theta)X(\theta) + B_eY(\theta)\right\} - \dot{X}(\theta) < 0 \tag{15}$$

$X(\theta)$ can be seen as Lyapunov matrix for the dimension of the state space representation. In Eq.(14), there are products of scheduling parameters and Lyapunov matrix $X_{lf}(\theta)$. $X_{lf}(\theta)$ is assigned as form of Eq.(16) and restricted as Eq.(17). The Eq.(14) become multi-affine for θ. The stability is guaranteed by solving finite LMI conditions at vertexes of the scheduling parameter θ and derivative $\dot{\theta}$.

$$X_{lf}(\theta) = X_{lf0} + \sum_{i=1}^{3}\theta_i X_{lfi} \tag{16}$$

$$X_{lfi} = \begin{bmatrix} X_{1.1-i} & \dots & X_{1.5-i} & 0 & 0 \\ & & \vdots & & \\ X_{5.1-i} & \dots & X_{5.5-i} & 0 & 0 \\ X_{6.1-i} & & \dots & & X_{6.10-i} \\ & & \vdots & & \\ X_{10.1-i} & & \dots & & X_{10.10-i} \end{bmatrix}$$

$$\begin{aligned} &(i = 1, ..., 3) \\ &\theta_1[X_{k.1-1} \dots X_{k.10-1}] = 0 \ (k = 7, 8) \qquad (17) \\ &\theta_2[X_{9.1-2} \dots X_{9.10-2}] = 0 \\ &\theta_3[X_{10.1-3} \dots X_{10.10-3}] = 0 \end{aligned}$$

3.3 LQ control specification

For Eq.(9), an evaluated function J_z is given by Eq.(18).

$$J_z = \int_0^\infty ({x_e}^{\mathrm{T}}Qx_e + u^{\mathrm{T}}Ru)dt \tag{18}$$

Where, $Q \geq 0$ and $R > 0$. Matrices $A_{lf}(\theta)$, $X_{lf}(\theta)$, $X(\theta)$, $Y_{lf}(\theta)$, $Y(\theta)$ and $\dot{X}_{lf}(\theta)$ are affine to the scheduling parameter and its derivative. Matrices can be represented by parameter box Θ_i.

$$\Theta_1 = (\underline{\theta}_0, \underline{\theta}_1, \underline{\theta}_2, \underline{\theta}_3, \underline{\theta}_4), \Theta_2 = (\overline{\theta}_0, \underline{\theta}_1, \underline{\theta}_2, \underline{\theta}_3, \underline{\theta}_4), ...$$
$$...\Theta_{31} = (\underline{\theta}_0, \overline{\theta}_1, \overline{\theta}_2, \overline{\theta}_3, \overline{\theta}_4), \Theta_{32} = (\overline{\theta}_0, \overline{\theta}_1, \overline{\theta}_2, \overline{\theta}_3, \overline{\theta}_4)$$

For stabilizing the LPV system Eq.(10), LMI conditions which derive GS controller minimize evaluated function J_z from w to z can be obtained as Eq.(19)-(22).

$$\begin{aligned} &\text{minimize} \ : \ \gamma \\ &\text{subject to} \\ &X(\Theta_i) > 0 \qquad (19) \\ &\begin{bmatrix} \mathrm{He}\{A_{lf}(\Theta_i)X_{lf}(\Theta_i) + B_{lf}Y_{lf}(\Theta_i)\} - E_{lf}\dot{X}_{lf}(\Theta_i) & Y_{l}f^{\mathrm{T}}(\Theta_i)R^{\frac{1}{2}T} & X_{l}f(\Theta_i)Q_{lf}^{\frac{1}{2}} \\ R^{\frac{1}{2}}Y_{lf}(\Theta_i) & -I_{lf1} & 0 \\ Q_{lf}^{\frac{1}{2}}X_{lf}(\Theta_i) & 0 & -I_{lf2} \end{bmatrix} < 0 \qquad (20) \\ &\begin{bmatrix} W & I_{lf3} \\ I_{lf3} & X(\Theta_i) \end{bmatrix} > 0 \qquad (21) \\ &\mathrm{Trace}(W) < \gamma^2 \qquad (22) \\ &(i = 1, ..., 32) \end{aligned}$$

Where, I_{lf1}, I_{lf2}, I_{lf3} are unit matrices which have the appropriate dimensions. $X(\theta)$ and $Y(\theta)$ are obtained by $X(\Theta_i)$, $Y(\Theta_i)$ that satisfy Eq.(19)-(22). The GS controller $K(\theta)$ is assigned as $K(\theta) = [Y(\theta) \quad X(\theta)^{-1}]$.

3.4 Estimation of derivative

In Eq.(19)-(22), GS controller via parameter dependent Lyapunov function is synthesized. However, there is a Lyapunov matrix derivative. The controller guarantees robustness of variation of $\ddot{i}_p$ which does not include in dynamics. A estimation method of Lyapunov matrix derivative is adopted[5]. Matrix polytopes are define as Eq.(23).

$$\begin{aligned} &Co[\alpha](M_1, ..., M_q) \\ &:= \left[\sum_{i=1}^{q}\alpha_i M_i : \alpha_i \geq 0, \sum_{i=1}^{q}\alpha_i = 1\right] \end{aligned} \tag{23}$$

Let upper bound of Lyapunov matrix $\overline{X}_0$ be Eq.(24).

$$\text{For vertexes of } \dot{\alpha}_i < 0,\ 0 \leq X_i \leq \overline{X}_0,\ \forall_i \in I_D \tag{24}$$

Where, I_D is set of indexes of $\dot{\alpha}_i < 0$. $\dot{X}$ is guaranteed by an upper bound of Lyapunov function as Eq.(25).

$$\begin{aligned} -\dot{X} &= \sum_{i \in I_I}(-\dot{\alpha}_i X_i) + \sum_{i \in I_D}(-\dot{\alpha} X_i) \\ &\leq \mu_0 \overline{X}_0 \end{aligned} \tag{25}$$

Where, μ_0 is $\max\sum_{i \in I_D}(-\dot{\alpha})$. $\alpha_i (i = 1, ..., 8)$ are derived as Eq.(26).

$$\begin{aligned} \alpha_1 &= \frac{(\overline{\theta}_1 - \theta_1)(\overline{\theta}_2 - \theta_2)(\overline{\theta}_3 - \theta_3)}{(\overline{\theta}_1 - \underline{\theta}_1)(\overline{\theta}_2 - \underline{\theta}_2)(\overline{\theta}_3 - \underline{\theta}_3)} \\ \alpha_2 &= \frac{(\theta_1 - \underline{\theta}_1)(\overline{\theta}_2 - \theta_2)(\overline{\theta}_3 - \theta_3)}{(\overline{\theta}_1 - \underline{\theta}_1)(\overline{\theta}_2 - \underline{\theta}_2)(\overline{\theta}_3 - \underline{\theta}_3)} \\ &\vdots \\ \alpha_8 &= \frac{(\theta_1 - \underline{\theta}_1)(\theta_2 - \underline{\theta}_2)(\theta_3 - \underline{\theta}_3)}{(\overline{\theta}_1 - \underline{\theta}_1)(\overline{\theta}_2 - \underline{\theta}_2)(\overline{\theta}_3 - \underline{\theta}_3)} \end{aligned} \tag{26}$$

$\dot{X}_{lf}$ in Eq.(20) is shown as Eq.(12). We should derive upper bound of X. LMI conditions which include the estimation method of Lyapunov matrix derivative are shown as Eq.(27)-(31).

$$\begin{aligned}
&\text{minimize} \ : \ \gamma \\
&\text{subject to} \\
&\overline{X}_0 - X(\Theta_i) > 0 && (27)\\
&X(\Theta_i) > 0 && (28)\\
&\begin{bmatrix} \mathrm{He}\{A_{lf}(\Theta_i)X_{lf}(\Theta_i) + B_{lf}Y_{lf}(\Theta_i)\} + \mu_0\overline{X}_0 & Y_{lf}{}^{\mathrm{T}}(\Theta_i)R^{\frac{1}{2}T} & X_{lf}(\Theta_i)Q_{lf}^{\frac{1}{2}} \\ R^{\frac{1}{2}}Y_{lf}(\Theta_i) & -I_{lf1} & 0 \\ Q_{lf}^{\frac{1}{2}}X_{lf}(\Theta_i) & 0 & -I_{lf2} \end{bmatrix} < 0 && (29)\\
&\begin{bmatrix} W & I_{lf3} \\ I_{lf3} & X(\Theta_i) \end{bmatrix} > 0 && (30)\\
&\mathrm{Trace}(W) < \gamma^2 && (31)\\
&(i = 1, ..., 16)
\end{aligned}$$

4 Simulations

Upper and lower bounds of the varying parameters are assigned as $\theta_0 \in [0.147, 1.470]$, $\theta_1 \in [0.1, 0.7]$, $\theta_2 \in [-0.23, 0.23]$, $\theta_3 \in [-0.292, 0.292]$, $\theta_4 \in [-1.3745, 1.3745]$. Results of controllers are shown by several simulations. The reference is assigned 1.0 [m] for the horizontal position y of the load with hoisting movement. The reference is assigned 1.0 [m] for the horizontal position y of the load with hoisting movement. Results of input and output considering variation of the mass of the load and considering fixed mass ($m_p = 0.735$ [kg]) are shown in Fig. 2 and 3. These controllers are not adopted estimation of derivative and are assigned the weight matrix as $Q = [1, 1, 1, 1, 1], R = 1$. From Fig. 2 and 3, the oscillation is eliminated by con-

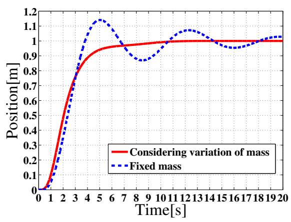

Figure 2 output: considering mass $m_p = 1.47$

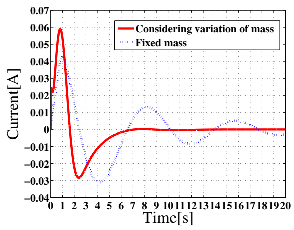

Figure 3 input: considering mass, $m_p = 1.47$

sidering variation of the mass of the load. Results of input and output adopting descriptor representation, LFT and adopting only descriptor representation are shown in Fig. 4 and 5. The Fig. 4 and 5 are not adopted estimation of derivative and are assigned the weight matrix as $Q = [1, 1, 1, 1, 1], R = 10$, the mass of the load as 0.147. γ achieved by adopting LFT and descriptor representation is 6.87, only descriptor representation is 7.23. From Fig. 5 and γ, the controller adopting LFT and descriptor representation can suppress the control input and make upper bound of evaluated function lower than adopting only descriptor representation. Results of input and output adopting estimation of derivative and not adopting estimation of derivative are shown in Fig. 6 and 7. The Fig. 6 and 7 are not considering $\dot{l}_p, \ddot{l}_p$ and are assigned the weight matrix as $Q = [10, 1, 1, 10, 1], R = 10$, the mass of the load

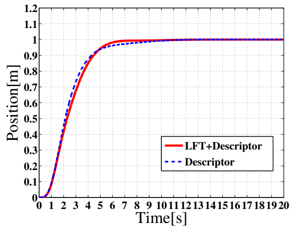

Figure 4 output: LFT and Descriptor

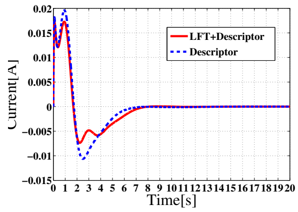

Figure 5 input: LFT and Descriptor

as 0.147. γ achieved by adopting estimation of derivative is 10.9, not adopting the method is 11.6. From

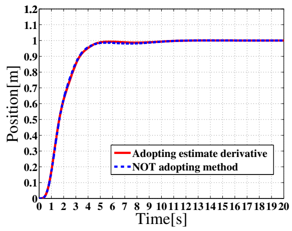

Figure 6 output: estimation of derivative θ_1

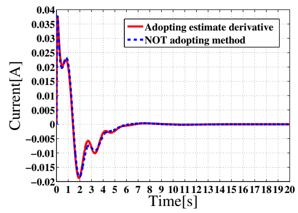

Figure 7 input: estimation of derivative θ_1

Fig. 7 and γ, the controller adopting estimation of Lyapunov matrix derivative can suppress the control input and make upper bound of evaluated function lower than not adopting.

5 Conclusion

This paper synthesizes Gain-Scheduling control for crane which guarantees variation of the mass of the load, rope length, its velocity and acceleration. The controller is synthesized with parameter dependent Lyapunov function. For improve performance of controller, descriptor representation, Linear Fractional Transformation and estimation of Lyapunov matrix derivative are adopted. By several simulations, it is shown that these methods are effective for the crane.

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