Nonlinear Control for First-Order Nonholonomic System with Hardware Restriction and Disturbance

M2013SC017 Shota WASHIZU
Supervisor : Isao TAKAMI

1 Introduction

After the 1990s, various control methods of nonholonomic systems are proposed. Nonholonomic systems have constraint conditions containing not only angles but also angular velocities and angular accelerations of generalized coordinates. Also these systems have outputs more than inputs, which are called under-actuated system. The linear control theory is hard to apply because linear approximated systems at the equilibrium point of these systems are not ensured the controllability. Additionally it is proven that systems cannot be stabilized using by linear time-invariant state feedback even if the controllability of systems are ensured in the meaning of nonlinear [1]. The canonical system exists for first-order nonholonomic systems which are represented by a symmetric affine system. The canonical system is called chained system [2]. Various feedback control strategies based on the chained system have been applied because of its adaptability.

Control moment gyroscope (CMG), which is treated in this research, is applied for attitude control of a large scale spacecraft, for example International Space Station. CMG provides larger torque than a conventional device by tilting the gimbal which is attached in the same axis as spinning rotor. CMGs are generally applied in cooperation [3]. In this research, the control for single CMG is discussed as a fundamental researches before discussing a cooperation control of CMGs. CMG is the first-order nonholonomic system and its state equation is represented by a 2-input 3-state symmetric affine system which has non-integrable constraint for angular velocities. In addition, CMG has the hardware restriction of the motion range and singularity. Recently efficient methods are proposed for the single CMG, for example the backstepping control for the chained system [4], the optimal feedback control based on the output regulation [5].

In this research, the tracking controller for CMG with nonholonomic constraint is synthesized. Firstly the mathematical model of CMG is derived. It is well known that a coulomb friction exists in CMG with this research [6]. However the influence of the friction is ignored in the mathematical model because the chained system becomes complicated if the influence of the friction is considered. Secondly the chained system is derived by the mathematical model. The chained system is obtained by a conversion method satisfying the hardware restriction. Thirdly a tracking controller based on the backstepping approach is synthesized. The influence of the friction is ignored in a mathematical model. The difficulty to thereby occur is dealt with the controller with a integrator. The stability of the system including the integrator is guaranteed theoretically based on Lyapunov function. Lastly the effectiveness of this research is illustrated by simulations. The controller with integrator is compared with the controller without integrator.

2 Mathematical Model

In this section, the mathematical model of CMG is derived. Firstly the equation of motion of CMG is derived from Euler-Lagrange [7]. Secondly the hardware restriction and singularity are described. Lastly the state equation which is obtained by the constraint condition is converted to the chained system [4].

2.1 Equation of Motion

The schematic diagram of CMG (Model-750 unit by Educational Control Products) is shown in Fig.1. CMG consists of 3 rigid bodies which are Rotor1, Gimbal2 and Gimbal3. Rotor1 and Gimbal2 are driven by DC motors. A coulomb friction exists in Rotor1 [6]. Gimbal2 has the range of motion $-\frac{\pi}{2} < q_2 < \frac{\pi}{2}$ [7] and singularity $q_2 = 0$ in the controller synthesis (details later). Synthesizing the controller has to be considered under the following restriction of the angle of Gimbal2.

$$0 < q_2 < \frac{\pi}{2} \tag{1}$$

Note that Gimbal3 does not have any drive sources. Gimbal3 is driven by the gyro torque which is generated by the law of conservation of angular momentum. The control problem of the research is that the angle of Gimbal3 tracks the reference without error in case that a coulomb friction exists in Rotor1. The chained system becomes complicated because the state equation can not be represented as symmetric affine system if the mathematical model has the friction. Therefore the influence of the friction is ignored in the mathematical model. The influence of the friction is dealt with the controller with integrator.

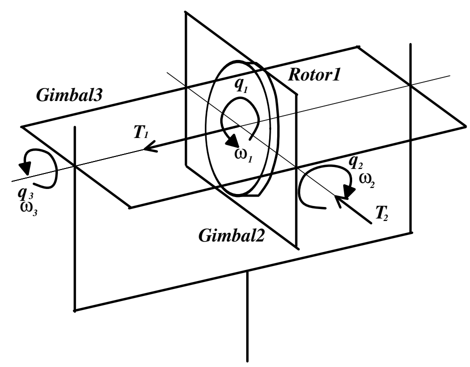

Figure 1 Schematic Diagram of CMG

Let q_i and $\omega_i, (i = 1, 2, 3)$ be the angle and the angular velocity of Rotor1, Gimbal2 and Gimbal3 respectively. T_1 and T_2 are the torque spinning Rotor1 and tilting Gimbal2. Equation of motions of each rigid bodies are obtained as follows.
Rotor1:

$$I_{R1y}\dot{\omega}_1 + I_{R1y}\dot{\omega}_3 \cos q_2 - I_{R1y}\omega_2\omega_3 \sin q_2 = T_1 \tag{2}$$

Gimbal2:

$$(I_{G2x}+I_{R1x})\dot{\omega}_2+I_1\omega_3^2\sin q_2\cos q_2 + I_{R1y}\omega_1\omega_3\sin q_2 = T_2 \quad (3)$$

Gimbal3:

$$\begin{aligned}(I_2-I_1\sin^2 q_2)\dot{\omega}_3+I_{R1y}\dot{\omega}_1\cos q_2 \\ -I_{R1y}\omega_1\omega_2\sin q_2 - I_1\omega_2\omega_3\sin 2q_2 = 0, \quad (4) \\ I_1 = I_{G2y}+I_{R1y}-I_{G2z}-I_{R1x}, \\ I_2 = I_{G3y}+I_{G2y}+I_{R1y}\end{aligned}$$

Physical parameters are as follows.

I_{R1x}, I_{R1y} : Moment of inertia of Rotor1$[\mathrm{kg}\cdot\mathrm{m}^2]$,

$I_{G2x}, I_{G2y}, I_{G2z}$: MOI of Gimbal2$[\mathrm{kg}\cdot\mathrm{m}^2]$,

I_{G3y} : MOI of Gimbal3$[\mathrm{kg}\cdot\mathrm{m}^2]$

2.2 Conversion into the Chained System

Consider the state equation of Gimbal3 to derive the chained system. The case that the initial angular momentum of Gimbal3 is zero is considered to derive the chained system simply (chaiend system becomes complicated if Gimbal3 has angular momentum at initial condition). Constraint equation of Gimbal3 is obtained by integral of equation (4) as follows.

$$(I_2-I_1\sin^2 q_2)\omega_3+I_{R1y}\omega_1\cos q_2 = 0 \quad (5)$$

Constraint equation (5) means the law of conservation of angular momentum. Equation (5) contains not only angles of the system but also angular velocities. This system is called the first-order nonholonomic system. Let $q=[q_1\ q_2\ q_3]^T$ be states of the state equation. From constraint equation (5), the following state equation is obtained by regarding ω_1 and ω_2 as inputs.

$$\dot{q} = \begin{bmatrix}1\\0\\\alpha(q_2)\end{bmatrix}\omega_1 + \begin{bmatrix}0\\1\\0\end{bmatrix}\omega_2, \quad (6)$$

$$\alpha(q_2) = \frac{-I_{R1y}\cos q_2}{I_2-I_1\sin^2 q_2}$$

System (6) is converted to the chained system by following coordinate and input conversions [2].

$$\begin{cases}x_1=q_1\\x_2=\alpha(q_2)\\x_3=q_3\end{cases}, \begin{cases}u_1=\omega_1\\u_2=\beta(q_2)\omega_2\end{cases}, \quad (7)$$

$$\beta(q_2)=\frac{\mathrm{d}}{\mathrm{d}q_2}\alpha(q_2)$$

It is well known that conversion (7) is the general method. However the problem caused by the hardware restriction occur in the controller synthesis (details later) if system (6) is converted into the chained system by general method (7). Therefore following conversions are applied in order to satisfy the problem [4].

$$\begin{cases}x_1=\alpha(q_2)\\x_2=q_1\\x_3=q_1\alpha(q_2)-q_3\end{cases}, \begin{cases}u_1=\beta(q_2)\omega_2\\u_2=\omega_1\end{cases} \quad (8)$$

The chained system is obtained as follows.

$$\begin{bmatrix}\dot{x}_1\\\dot{x}_2\\\dot{x}_3\end{bmatrix} = \begin{bmatrix}1&0\\0&1\\x_2&0\end{bmatrix}\begin{bmatrix}u_1\\u_2\end{bmatrix} \quad (9)$$

The nonlinear tracking controller is synthesized based on chained system (9) with conversion (8).

3 Controller Synthesis

In this section, the tracking controller for system (9) with conversion (8) to state equation (6) is synthesized. Let $x_r=[x_{1r}\ x_{2r}\ x_{3r}]^T$ be the tracking orbit and u_{1r}, u_{2r} be tracking inputs. The reference system is defined as follows.

$$\begin{bmatrix}\dot{x}_{1r}\\\dot{x}_{2r}\\\dot{x}_{3r}\end{bmatrix} = \begin{bmatrix}1&0\\0&1\\x_{2r}&0\end{bmatrix}\begin{bmatrix}u_{1r}\\u_{2r}\end{bmatrix} \quad (10)$$

Let $x_e=[x_{1e}\ x_{2e}\ x_{3e}\ s_3]^T$, $x_{ie}=x_i-x_{ir}$, $(i=1,2,3)$ be the error between the state and the reference. Let $s_3=\int(x_3-x_{3r})\mathrm{d}t$ be the integral of the error between the function of the angle of Gimbal3 and the reference. The integrator makes the system track the reference without error when the coulomb friction exists in the system. The error dynamics is obtained as follows.

$$\begin{cases}\dot{x}_{1e}=u_1-u_{1r}\\\dot{x}_{2e}=u_2-u_{2r}\\\dot{x}_{3e}=x_2(u_1-u_{1r})+x_{2e}u_{1r}\\\dot{s}_3=x_{3e}\end{cases} \quad (11)$$

Control strategy of stabilizing system (11) have following steps.

Step A. Separate dynamics (11) into two subsystems because stability of x_{3e} depends on stability of x_{1e}. Stabilizing x_{1e} by applying u_1 firstly.

Step B. Synthesize stabilizing controller for the other subsystem by applying remaining input u_2 based on the backstepping approach.

Step C. Calculate control torques T_1 and T_2 based on the backstepping approach from obtained inputs for the chained system.

3.1 Stabilize the Subsystem Δ_1

System of x_e is separated into following subsystems Δ_1 and Δ_2.

$$\Delta_1 : \dot{x}_{1e}=u_1-u_{1r} \quad (12)$$

$$\Delta_2 : \begin{cases}\dot{x}_{2e}=u_2-u_{2r}\\\dot{x}_{3e}=x_2(u_1-u_{1r})+x_{2e}u_{1r}\\\dot{s}_3=x_{3e}\end{cases} \quad (13)$$

At first, subsystem Δ_1 is stabilized by applying feedback as follows.

$$\dot{x}_{1e}=u_1-u_{1r}=-k_1x_{1e}, \quad k_1>0 \quad (14)$$

Subsystem Δ_2 can be rewritten as follows when u_1-u_{1r} converses to zero by equation (14)

$$\begin{cases}\dot{x}_{2e}=u_2-u_{2r}\\\dot{x}_{3e}=x_{2e}u_{1r}\\\dot{s}_3=x_{3e}\end{cases} \quad (15)$$

Subsystem Δ_2 is stabilized by applying the remaining input u_2.

In the chained system by general method (7), stabilizing subsystem (12) means stabilizing the angle of Rotor1 firstly because $x_1=q_1$. In contrast, conversion method (8) stabilizes the function of q_2 firstly because $x_1=\alpha(q_2)$.

3.2 Stabilize the Subsystem Δ_2

Subsystem (15) is stabilized based on backstepping approach. The dynamics of x_{3e} and s_3 can be stabilized by x_{2e} which is regarded as the virtual input. Note that x_{2e} is defined as the error between the state x_2 and the reference x_{2r}. The error x_{2e} may become large because the dynamics of x_{3e} and s_3 is stabilized by regarding x_{2e} as the virtual input. In the case of general method (7), x_{2e} is defined by $x_2 = \alpha(q_2)$, i.e. x_{2e} is a function of the angle of Gimbal2. Gimbal2 has the restriction (1). It is difficult to satisfy restriction (1) with the general formed chained system. In contrast, the chained system by conversion (8) consists of $x_2 = q_1$. Synthesizing the controller becomes easy by applying conversion (8) because the angle of Rotor1 does not have any restrictions.

Consider the condition to stabilize the dynamics of x_{3e} and s_3 in the subsystem Δ_2 by regarding x_{2e} as the virtual input. Lyapunov function candidate $V_1(x_{3e}, s_3)$ for the dynamics of x_{3e} and s_3 is chosen as follows.

$$V_1(x_{3e}, s_3) = \frac{1}{2}(x_{3e} + u_{1r}^2 s_3)^2 > 0 \quad (16)$$

Assume the following equation is satisfied.

$$x_{2e} = -k_2 u_{1r}(x_{3e} + u_{1r}^2 s_3) - 2\dot{u}_{1r} s_3 - u_{1r} x_{3e}, \quad k_2 > 0 \quad (17)$$

Then the time derivative of equation (16) is calculated as follows.

$$\dot{V}_1(x_{3e}, s_3) = -k_2 u_{1r}^2 (x_{3e} + u_{1r}^2 s_3)^2 < 0 \quad (18)$$

The dynamics of x_{3e} and s_3 becomes asymptotically stable if assumption (17) is satisfied.

Consider the virtual input u_2 to satisfy assumption (17). The error σ between the state x_{2e} and right hand side of equation (17) is defined as follows.

$$\sigma = x_{2e} + k_2 u_{1r}(x_{3e} + u_{1r}^2 s_3) + 2\dot{u}_{1r} s_3 + u_{1r} x_{3e} \quad (19)$$

Lyapunov function candidate $V_2(x_{3e}, s_3, \sigma)$ for the dynamics of x_{3e}, s_3 and σ is chosen as follows.

$$V_2(x_{3e}, s_3, \sigma) = \frac{1}{2}(x_{3e} + u_{1r}^2 s_3)^2 + \frac{1}{2}\sigma^2 > 0 \quad (20)$$

The input u_2 is chosen as follows.

$$\begin{aligned} u_2 = u_{2r} &- (3\dot{u}_{1r} + k_2 \dot{u}_{1r} + k_2 u_{1r}^3) x_{3e} \\ &- (u_{1r}^2 + k_2 u_{1r}^2) x_{2e} - (2\ddot{u}_{1r} + 3k_2 u_{1r}^2 \dot{u}_{1r}) s_3 \\ &- u_{1r}(x_{3e} + u_{1r}^2 s_3) - k_3 \sigma, \quad k_3 > 0 \end{aligned} \quad (21)$$

The time derivative of equation (20) is calculated as follows.

$$\dot{V}_2(x_{3e}, s_3, \sigma) = -k_2 u_{1r}^2 (x_{3e} + u_{1r}^2 s_3)^2 - k_3 \sigma^2 < 0 \quad (22)$$

The dynamics of x_{3e}, s_3 and σ becomes asymptotically stable. Assumption (17) is satisfied by choosing u_2 as equation (21) because the dynamics of σ converges to zero. Therefore the subsystem Δ_2 becomes asymptotically stable.

3.3 Calculate the Control Torque

Control torques T_1 and T_2 are derived by regarding u_1 and u_2 as virtual inputs. The system of CMG is represented by equation (2)-(4) and equation (14) as follows.

$$\begin{cases} \dot{u}_1 = \beta(q_2)\dot{\omega}_2 + \gamma(q_2)\omega_2^2, \ \gamma(q_2) = \frac{\mathrm{d}\beta(q_2)}{\mathrm{d}q_2} \\ \dot{\omega}_2 = f_1(q_2, \omega_1, \omega_3) + \frac{1}{I_{G2x} + I_{R1x}} T_2 \\ \dot{x}_{1e} = u_1 - u_{1r} \end{cases} \quad (23)$$

$$\begin{cases} \dot{u}_2 = \dot{\omega}_1 = f_2(q_2, \omega_1, \omega_2, \omega_3) + f_3(q_2) T_1 \\ \dot{\sigma} = u_2 - u_{2r} + (3\dot{u}_{1r} + k_2 \dot{u}_{1r} + k_2 u_{1r}^3) x_{3e} \\ \quad +(u_{1r}^2 + k_2 u_{1r}^2) x_{2e} + (2\ddot{u}_{1r} + 3k_2 u_{1r}^2 \dot{u}_{1r}) s_3 \\ \dot{x}_{3e} = x_{2e} u_{1r} = \sigma u_{1r} - k_2 u_{1r}^2 (x_{3e} + u_{1r}^2 s_3) \\ \quad -2u_{1r}\dot{u}_{1r} s_3 - u_{1r}^2 x_{3e} \end{cases} \quad (24)$$

$$f_1(q_2, \omega_1, \omega_3) = \frac{-I_1 \omega_3^2 \sin q_2 \cos q_2 - I_{R1y}\omega_1\omega_3}{I_{G2x} + I_{R1x}},$$

$$f_2(q_2, \omega_1, \omega_2, \omega_3) = \frac{f_{2a}(q_2, \omega_1, \omega_2, \omega_3)}{-I_2 + I_1 \sin^2 q_2 + I_{R1y}\cos^2 q_2},$$

$$\begin{aligned} f_{2a}(q_2, \omega_1, \omega_2, \omega_3) = &-(I_2 - I_1 \sin^2 q_2)\omega_2\omega_3 \sin q_2 \\ &+ I_{R1y}\omega_1\omega_2 \cos q_2 \sin q_2 \\ &+ I_1 \omega_2\omega_3 \sin 2q_2 \cos q_2, \end{aligned}$$

$$f_3(q_2) = \frac{I_2 - I_1 \sin^2 q_2}{-I_{R1y}(I_2 - I_1 \sin^2 q_2) + I_{R1y}^2 \cos^2 q_2}$$

At first, the torque T_2 is derived by backstepping approach. The error ξ between the state u_1 and virtual input (14) is as follows.

$$\xi_1 = u_1 - (u_{1r} - k_1 x_{1e}) \quad (25)$$

Lyapunov function candidate $V_3(x_{1e}, \xi_1)$ for the dynamics of x_{1e} and ξ_1 is chosen as equation (26).

$$V_3(x_{1e}, \xi_1) = \frac{1}{2}x_{1e}^2 + \frac{1}{2}\xi_1^2 > 0 \quad (26)$$

The function $\dot{u}_1$ is chosen as follows.

$$\dot{u}_1 = -x_{1e} + \dot{u}_{1r} - k_1 \dot{x}_{1e} - H_1 \xi_1, \quad H_1 > 0 \quad (27)$$

Time derivative of equation (26) is calculated as follows.

$$\dot{V}_3(x_{1e}, \xi_1) = -k_1 x_{1e}^2 - H_1 \xi_1^2 < 0 \quad (28)$$

System (23) becomes asymptotically stable. Therefore the control torque T_2 is obtained from equation (23), (27) as follows.

$$\begin{aligned} T_2 = \{(I_{G2x} + I_{R1x})(-\beta(q_2) f_1 - \gamma(q_2)\omega_2^2 + \dot{u}_{1r} \\ -k_1 \dot{x}_{1e} - x_{1e} - H_1 \xi_1)\}/\beta(q_2) \end{aligned} \quad (29)$$

Note that $\beta(q_2) = 0$ in equation (29) is singularity. Control torque T_2 becomes infinity if q_2 goes to zero. In the same way, the torque T_1 is derived by backstepping approach. The error ξ_2 between the state u_2 and virtual input (21) is as follows.

$$\xi_2 = u_2 - u_{2r} + G_1 x_{2e} + G_2 x_{3e} + G_3 s_3 \quad (30)$$

$$G_1 = u_{1r}^2 + k_2 u_{1r}^2 + k_3,$$

$$G_2 = 3\dot{u}_{1r} + k_2 \dot{u}_{1r} + k_2 u_{1r}^3 + u_{1r} + k_3 u_{1r} + k_2 k_3 u_{1r},$$

$$G_3 = 2\ddot{u}_{1r} + 3k_2 u_{1r}^2 \dot{u}_{1r} + u_{1r}^3 + 2k_3 \dot{u}_{1r} + k_2 k_3 u_{1r}^3$$

Lyapunov function candidate $V_4(x_{3e}, s_3, \sigma, \xi_2)$ for the dynamics of x_{3e}, s_3, σ and ξ_2 is chosen as equation (31).

$$V_4(x_{3e}, s_3, \sigma, \xi_2) = \frac{1}{2}(x_{3e} + u_{1r}^2 s_3)^2 + \frac{1}{2}\sigma^2 + \frac{1}{2}\xi_2^2 > 0 \tag{31}$$

The function $\dot{u}_2$ is chosen as follows.

$$\begin{aligned} \dot{u}_2 = (&\dot{u}_{2r} - \dot{G}_1 x_{2e} - G_1 \dot{x}_{2e} - \dot{G}_2 x_{3e} - G_2 \dot{x}_{3e} \\ &\dot{G}_3 s_3 - G_3 \dot{s}_3 - \sigma - H_2 \xi_2)/f_3, H_2 > 0 \end{aligned} \tag{32}$$

Time derivative of equation (31) is calculated as follows.

$$\dot{V}_4(x_{3e}, s_3, \sigma, \xi_2) = -k_2 u_{1r}^2 (x_{3e} + u_{1r}^2 s_3)^2 - k_3 \sigma^2 - H_2 \xi_2^2 < 0 \tag{33}$$

System (24) becomes asymptotically stable. Therefore the control torque T_1 is obtained from equation (24), (32) as follows.

$$\begin{aligned} T_1 = (&-f_2 + \dot{u}_{2r} - \dot{G}_1 x_{2e} - G_1 \dot{x}_{2e} - \dot{G}_2 x_{3e} - G_2 \dot{x}_{3e} \\ &-\dot{G}_3 s_3 - G_3 \dot{s}_3 - \sigma - H_2 \xi_2)/f_3, H_2 > 0 \end{aligned} \tag{34}$$

Then the system of CMG becomes asymptotically stable by T_1 and T_2.

4 Simulation

In this section, the effectiveness of this research is illustrated by simulations. Simulations including a friction in Rotor1 are executed. The friction is estimated by some experiments as follows.

$$F_c = \begin{cases} 0.5, & \omega_1 > 0 \\ 0, & \omega_1 = 0 \\ -0.5, & \omega_1 < 0 \end{cases} \tag{35}$$

The equation of motion of Rotor1 with friction is as follows.

$$I_{R1y}\dot{\omega}_1 + I_{R1y}\dot{\omega}_3 \cos q_2 - I_{R1y}\omega_2\omega_3 \sin q_2 = T_1 - F_c \tag{36}$$

Initial conditions are given as follows.

$$\begin{aligned} \begin{bmatrix} q_1 & q_2 & q_3 \end{bmatrix}^T &= \begin{bmatrix} 0 & \frac{\pi}{4} & 0 \end{bmatrix}^T, \\ \begin{bmatrix} \omega_1 & \omega_2 & \omega_3 \end{bmatrix}^T &= \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T \end{aligned} \tag{37}$$

References are given as follows.

$$\begin{aligned} \omega_{1r} &= \begin{cases} 0 & (t < 5, t > 7) \\ -1.008 & (5 \le t \le 7) \end{cases} \quad \text{[rad/sec]}, \\ q_{2r} &= 0.5\sin(1.5t) + \frac{\pi}{4} \quad \text{[rad]}, \\ \omega_{3r} &= \omega_{1r}\alpha(q_2) \quad \text{[rad/sec]} \end{aligned} \tag{38}$$

Where q_{3r} is obtained by integral of ω_{3r}. The control problem is that Gimbal3, which does not have any drive sources, tracks the reference without error. The angle of Gimbal2 tracks the sinusoidal reference satisfying restriction (1). The reference of the angular velocity of Rotor1 ω_{1r} is calculated to let the reference for the angle of Gimbal3 go to 0.5[rad]. Gain parameters are chosen by trial and error as follows.

$$k_1 = 3, k_2 = 7, k_3 = 11, H_1 = 18, H_2 = 6 \tag{39}$$

Simulation results are shown in Fig.2 - 4. The solid line shows the proposed method (controller1) and the dashed line shows the controller without integrator (controller2). The dotted line shows the reference.

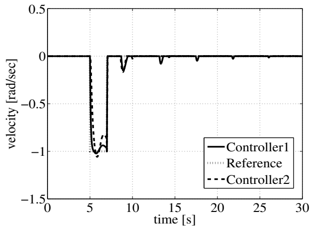

Figure 2 Simulation of ω_1

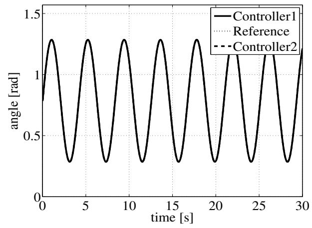

Figure 3 Simulation of q_2

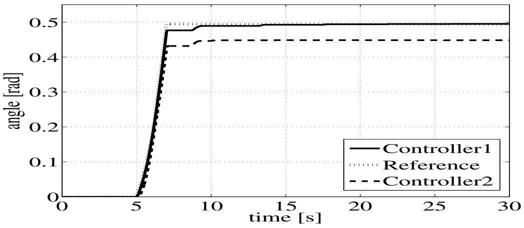

Figure 4 Simulation of q_3

The angular velocity of Rotor1 is shown in Fig.2. It can be seen that the response ω_1 is stabilized. The angle of Gimbal2 with the restriction is shown in Fig.3. As can be seen that the response q_2 tracks the reference under the restriction (1). The angle of Gimbal3 is shown in Fig.4. The steady-state error occurs in the response q_3 applying for the controller2. In contrast, the response q_3 tracks the reference without error by applying proposed method in case that the friction exists in the system. From these results, the control problem is satisfied by applying the proposed method.

5 Conclusion

In this research, a nonlinear tracking control for first-order nonholonomic system with restriction and friction is proposed. The state equation is converted to the chained system which avoids the restriction. The tracking controller with integrator based on the backstepping method is synthesized. The integrator makes states track the reference without error when the coulomb friction exists in the system. The stability of the system with integrator is guaranteed theoretically by consisting Lyapunov function. From simulation results, it can be seen that the response can track the reference without error by the proposed method in case that the system has restriction and friction.

References

[1] R. W. Brockett: Asympototic stability and feedback stabilization. Differential Geometric Control Theory, pp. 181-191, Boston, Birkhauser, 1983.

[2] F. Matsuno and J. Tsurusaki: Chained form transformation algorithm for a class of 3-states and 2-inputs nonholonomic systems and attitude control of a space robot. Proc. the 38th IEEE CDC, pp. 2126-2131, 1999 .

[3] S. Sato and M. Takahashi: Singularity Escape of Satellite with Control Moment Gyros Using Path Planning of Angular Momentum. The Japan Society of Mechanical Engineers, Vol. 79, No. 801, pp. 1526-1539, 2013.

[4] M. Reyhanoglu and J. van de Loo: State Feedback Tracking of a Nonholonomic Control Moment Gyroscope. Proc. the 45th IEEE CDC, pp. 6156-6161, 2006.

[5] K. Ishikawa, N. Sakamoto and D. Shiraki: Design of partial optimal controller for a nonlinear system and application to the attitude control of a control moment gyro. Proc. SICE Annual Conference (SICE), pp. 1865-1871, 2013.

[6] S. Yang: Adaptive Control for Double-Integrator Class Systems in the Absence of Velocity Feedback. The University of Texas at Austin, Aerospace Engineering and Engineering Mechanics Department, Austin, TX, 2012.

[7] T. R. Parks: Manual For Model 750 Control Moment Gyroscope. Educational Control Products, Bell Canyon, 1999.