Robust LQ Control for Biped Robot based on Inverted Pendulum

M2014SC002 Masahiro HAYASHI
supervisor : Isao TAKAMI

Abstract

In this study, the control method for biped robot walking system is proposed. Furthermore, the robust stability for length of legs is guaranteed by using polytopic representation. Descriptor representation and linear fractional transformation(LFT) are adopted to obtain an equivalent polytopic representation of biped robot walking system which has multi affine. The robust controller is obtained by solving a finite set of Linear Matrix Inequalities (LMIs). The robust LQ controller that guarantees the stability of the system for variation range of length of leg is designed. The effectiveness of proposed method is illustrated by simulations.

1 Introduction

Recently, several robots are used in various places. In several robots, biped robot has mainly two strong points that flexibility and sense of affinity. Everything that exist in the world is made on the premise that used by human. The usage environment also is similar. It is assumed that Biped robot that resembles to human soon accommodates itself to the environment. Because biped robot is similar, it is assumed that human is easy to find something congenial. Thus, it is assumed that these features are used in the care, danger area which cannot enter and so on. In the danger area, if there is an accident, biped robot soon accommodates itself it and can keep action. In the care, from its visual aspect, requiring long-term care feel as if they are received the care by human doing. Thus, it is expected that biped robot is used as substitute for human. To use biped robot as substitute for human, it is necessary for it to perform like human. It is mentioned that walking like human is important point to achieve the performance. To achieve the walking, many researchers studied many researches. For example, Qiang Huang et al. (2001) studied about an actuator used link[1], Kemalettin Erbatur et al. (2009) studied about many link walking with 12 DOF[2]. Many of these studies adopted manipulator as foot. However, it is a strong nonlinear and complex system. It is not easy to control it. It takes moderate times to achieve controlling it, which is difficult to perform like human.

In this study, by using simple system, biped walking is achieved easily and more quickly. In generally, biped robot has two legs so that it can walk by changing one leg called support leg and the other leg called swing leg. The support leg is regard as inverted pendulum. In many research, swing leg is regarded as simple pendulum from its dynamics. However, the simple pendulum depends on gravity so that it is not controlled and it takes some times. To walk quickly, the swing leg is also regarded as inverted pendulum that is enabled to control itself from its dynamics. In this study, because linear inverted pendulum is enabled to keep waist position constant by stretching itself so that biped walking is used easily, it is adopted as inverted pendulum in each leg. To walk easily, biped robot is assumed only to leave two legs. However, biped robot is similar visual aspect of human so that mass and length of robot also are required to the parameters which is similar those of human. Length of legs is varied by biped robot of type and swing leg is stretched during it is moved down so that length of leg is varying parameter. The dynamics of biped robot depends on the varying parameter. The robust stability is required to design biped robot with varying parameter. By using matrix polytopic representation, the robust stability for the system has varying parameter is guaranteed theoretically. Descriptor representation and linear fractional transformation(LFT) are adopted to obtain an equivalent polytopic representation of biped robot walking system which has multi affine. The problem is formulated as solving a finite set of Linear Matrix Inequalities(LMIs). In this study, to achieve to walk similar human, it is also required the walking speed so that LQ controller that yields an optimum performance with small input and high readiness by minimizing the cost function is used. Then, this study adopts the robust LQ controller that guarantees the stability of the system for variation range of length of leg. Thus, by changing two inverted pendulums that guaranteed varying parameter, this study indicates that biped walking is achieved easily and quickly. At last, the effectiveness of proposed method is illustrated by simulations.

2 Modeling and Control Target

In this study, the biped robot in XZ plane is compared to an inverted pendulum, and a system is disassembled by handling as a simply system consists of a support leg and a swing leg as follows. Here, linear inverted pendulum which length from the floor to the top is a height of the robot's waist is applied as support leg. And a mass of an upper body of the robot is extremely large toward that of the support leg so that a gravity position of support leg exists the height of the robot 's waist. Moreover, the support leg and the swing leg are fixed on the floor and on the waist respectively. Here, when the support leg inclines to one side, a changed gravity position can go back to a prior height by expanding and contracting the inverted pendulum. By keeping the gravity position, walking of biped robot is considered easily.

2.1 Support leg

2.1.1 Control target and physical parameter

The model of the support leg used in this study is shown in Fig.1. Table 1 shows the physical constants and variables used in this study.

2.1.2 Dynamical equation

The mathematics model expresses behavior of support leg is derived as follows by using Newton 's motion

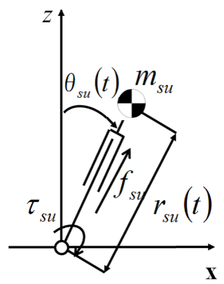

Fig. 1 Support leg

Table 1 Physical Constants and Variables

symbol	parameter	unit
g	Acceleration of gravity	$[\frac{kg}{s^2}]$
m_{su}	Mass of support leg	[kg]
f_{su}	Kick force of support leg	[N]
θ_{su}	Angle of support leg	[rad]
r_{su}	Length of support leg	[m]
τ_{su}	Torque to incline support leg	[Nm]

equation.

$$r_{su}(t)\ddot{\theta}_{su}(t) = \frac{\tau_{su}(t)}{m_{su}} - 2r_{su}(t)\dot{r}_{su}(t)\dot{\theta}_{su}(t) + gr_{su}(t)\sin\theta_{su}(t) \quad (1)$$

$$\ddot{r}_{su}(t) = \frac{f_{su}(t)}{m_{su}} + r_{su}(t)\dot{\theta}_{su}(t) - g\cos\theta_{su}(t) \quad (2)$$

To linear Eq.(2) and Eq.(2), let $r_{su}(t)$, $\theta_{su}(t)$ and $f_{su}(t)$ define as folllows.

$$\begin{cases} r_{su}(t) = r_{0su} + \delta r_{su}(t) \\ \theta_{su}(t) = \theta_{0su} + \delta\theta_{su}(t) \\ f_{su}(t) = f_{0su}(t) + \delta f_{su}(t) \end{cases} \quad (3)$$

where δr_{su}, $\delta\theta_{su}$ and $\delta f_{su}(t)$ are small variation of r_{su}, θ_{su} and f_{su} and r_{0su}, θ_{0su} and f_{0su} are initial value of r_{su}, θ_{su} and f_{su} and r_{0su}, θ_{0su} respectively. From Fig.1 a start position of inverted pendulum is stand upright so that θ_{0su} and $f_{0su}(t)$ are as follows.

$$\begin{cases} \theta_{0su} = 0 \\ f_{0su}(t) = m_{su} g \cos\theta_{su}(t) \end{cases} \quad (4)$$

And assuming that reference rake is within 30[deg], a linearization is obtained $\sin\theta_{su}(t) \approx \theta_{su}(t), \cos\theta_{su}(t) \approx 1$. By using Eq.(3) and Eq.(4), Eq.(1) and Eq.(2) are linearized as follows.

$$r_{0su}^2\delta\ddot{\theta}_{su}(t) = \frac{\tau_{su}(t)}{m_{su}} + gr_{0su}\delta\theta_{su}(t) \quad (5)$$

$$\delta\ddot{r}_{su}(t) = \frac{\delta f_{su}(t)}{m_{su}} \quad (6)$$

By let state variable $x_{su}(t) = [x_{1su}(t) \;\; x_{2su}(t) \;\; x_{3su}(t) \;\; x_{4su}(t)]^T = [\delta\theta_{su}(t) \;\; \delta\dot{\theta}_{su}(t) \;\; \delta r_{su}(t) \;\; \delta\dot{r}_{su}(t)]^T$ be and control input $u_{su}(t) = [\tau_{su}(t) \;\; \delta f_{su}(t)]^T$ be as follows, from Eq.(5) and Eq.(6), State space representation is obtain as follows.

$$\dot{x}_{su}(t) = A_{su}x_{su}(t) + B_{su}u_{su}(t) \quad (7)$$

$$y_{su}(t) = C_{su}x_{su}(t) \quad (8)$$

2.1.3 Extended system

To track output to reference without error, let that system extended as follows by let $x_{esu}(t) = [x_{su}(t) \;\; \omega_{su}(t)]^T = [x_{su}(t) \;\; \omega_{1su}(t) \;\; \omega_{2su}(t)]^T = [x_{su}(t) \;\; \int(\theta_{suref} - \delta\theta_{su})dt \;\; \int(r_{suref} - \delta r_{su})dt]^T$ be.

$$\dot{x}_{esu}(t) = A_{esu}x_{esu}(t) + B_{esu}u_{su}(t) + I_{esu}H_{esu} \quad (9)$$

$$A_{esu} = \begin{bmatrix} A_{su} & 0 \\ -C_{su} & 0 \end{bmatrix}, B_{esu} = \begin{bmatrix} B_{su} \\ 0 \end{bmatrix}, I_{esu} = \begin{bmatrix} 0 \\ I \end{bmatrix}, H_{esu} = \begin{bmatrix} \theta_{suref} \\ r_{suref} \end{bmatrix}$$

2.1.4 Polytopic representation

I'll show the process to obtain polytopic representation of the system with varying parameter r_{0su}. Because Eq.(9) has rational terms of varying parameter, it is difficult to use polytopic representation directly. The system is transformed to an equivalent system that is multi-affine r_{0su} by using descriptor representation and linear fractional transformation (LFT).

2.1.5 Transformation to polynomial

Eq.(9) has rational terms of r_{0su}. It is difficult to use polytopic representation. Eq.(9) is transformed to descriptor form to transform from rational to polynomial. Descriptor equation is obtained as follows.

$$E_{su}\dot{x}_{su}(t) = D_{su}x_{su}(t) + F_{su}u_{su}(t) \quad (10)$$

Eq.(10) is polynomial for varying parameters r_{0su}.

2.1.6 Putting Varying Parameter into One Matrix

Eq.(10) becomes polynomial for varying parameter r_{0su}. However, there exists this parameter in matrices E_{su} and D_{su}. If Eq.(10) is transformed to an ordinary state space representation $\dot{x}(t) = D_{su}^{-1}F_{su}x(t)$, matrices $E_{su}^{-1}D_{su}$ has rational terms of r_{0su}. It is difficult to use polytopic representation. I put varying parameter into one matrix by expanding dimension of matrices. Varying parameters r_{0su} is put into matrix D_dsu by defining $x_{dsu}(t) = [x_{esu}(t) \;\; \delta\ddot{\theta}_{su} \;\; \delta\ddot{r}_{su} \;\; \dot{\omega}_{su}(t) \;\; u_{su}(t)]^T$.

$$E_{dsu}\dot{x}_d(t) = D_{dsu}x_d(t) + F_{dsu}u(t) \quad (11)$$

There exist the varying parameter r_{0su}.

2.1.7 Transformation to Multi-affine

Only the matrix D_{dsu} has varying parameter. However there are higher oder terms of r_{0su} in it. In this case, it is still difficult for the matrix to use the polytopic representation. LFT is applied to tranform the high order terms of r_{0su} to first order terms of r_{0su}. Matrix D_{dsu} can be represented by Eq.(12). Here D_{nsu} is the matrix which only contains first order terms of r_{0su}. And $B_{\delta su}\Delta C_{\delta su}$ is the matrix which contains high order terms. D_{nsu}, $B_{\delta su}$, $C_{\delta su}$ and Δ_{su} are as follows.

$$D_{dsu} = D_{nsu} + B_{\delta su}\Delta_{su}C_{\delta su} \quad (12)$$

D_{nsu} and $C_{\delta su}$ are multi-affine with respect to m_{su} and r_{0su}, and $B_{\delta su}$ is constant matrix. Eq.(12) is expressed as follows by using D_{nsu}, $B_{\delta su}$, $C_{\delta su}$ and Δ_{su}.

$$\begin{aligned} E_{dsu}\dot{x}_d &= D_{nsu}x_d + B_{\delta su}w_{\delta su} + F_{dsu}u \quad (13) \\ z_{\delta su} &= C_{\delta su}x_{dsu} \\ w_{\delta su} &= \Delta z_{\delta su} \end{aligned}$$

Let $\tilde{x}_{dsu}$ be new descriptor variable $\tilde{x}_{dsu}(t) = [x_{dsu}^T(t) \quad z_{\delta su}(t)]^T$, and the descriptor equation which is transformed by LFT is as follows. Then, the system becomes multi-affine for m_{su} and r_{0su}.

$$\tilde{E}_{dsu}\dot{\tilde{x}}_{dsu}(t) = \tilde{D}_{dsu}\tilde{x}_{dsu}(t) + \tilde{F}_{dsu}u(t) \qquad (14)$$

$$\tilde{E}_{dsu} = \begin{bmatrix} E_{dsu} & 0 \\ 0 & 0 \end{bmatrix}, \tilde{F}_{dsu} = \begin{bmatrix} F_{dsu} \\ 0 \end{bmatrix}, \tilde{D}_{dsu} = \begin{bmatrix} D_{nsu} & B_{\delta su}\Delta_{su} \\ C_{\delta su} & -I \end{bmatrix}$$

D_{nsu}, $C_{\delta su}$ and $B_{\delta su}\Delta_{su}$ have not polynomial but only multi-affine terms of m_{su} and r_{0su}. Biped robot system which is multi-affine for m_{su} and r_{0su} can finally be obtained. In this study, the polytopic representation is used to guarantee the robustness. The upper and lower bound of r_{0su} composed of matrix A_{esu} and B_{esu} are as follows.

$$r_{0su} \in [r_{0sumin}, \ r_{0sumax}] \qquad (15)$$

Let $\tilde{D}_{d1}$ and $\tilde{D}_{d2}$ be the vertex matrices for the variation range of matrix $\tilde{D}_{dsu}$.

$$\tilde{D}_{d1} = \tilde{D}_{dsu}(r_{0sumin}), \ \tilde{D}_{d2} = \tilde{D}_{dsu}(r_{0sumax}) \quad (16)$$

It is assumed that the system stabilizes in the range of the convex hull of the two vertexes.

2.2 Swing leg

2.2.1 Control target and physical parameter

The model of swing leg used in this study is shown in Fig.2. Table 2 shows the physical constants and variables used in this study.

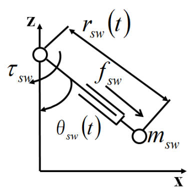

Fig. 2 Swing leg

Table 2 Physical Constants and Variables

symbol	parameter	unit
g	Acceleration of gravity	$[\frac{kg}{s^2}]$
m_{sw}	Mass of swing leg	[kg]
f_{sw}	Stretch force of swing leg	[N]
θ_{sw}	Angle of swing leg	[rad]
r_{sw}	Length of swing leg	[m]
τ_{sw}	Torque to swing up downward	[Nm]

2.2.2 Dynamical equation

The mathematics model expresses behavior of swing leg is derived as follows by using Newton's motion equation.

$$r_{sw}(t)\ddot{\theta}_{sw}(t) = \frac{\tau_{sw}(t)}{m_{sw}} - 2r_{sw}(t)\dot{r}_{sw}(t)\dot{\theta}_{sw}(t) \quad (17)$$
$$-r_{sw}(t)\sin\theta_{sw}(t)$$
$$\ddot{r}_{sw}(t) = \frac{f_{sw}(t)}{m_{sw}} + r_{sw}(t)\dot{\theta}_{sw}(t) \quad (18)$$
$$+g\cos\theta_{sw}(t)$$

To linear Eq.(18) and Eq.(19), let $r_{sw}(t)$, $\theta_{sw}(t)$ and $f_{sw}(t)$ define as with Eq.(3). Here, δr_{sw}, $\delta\theta_{sw}$ and $\delta f_{sw}(t)$ are small variation of r_{sw}, θ_{sw} and f_{sw} respectively. From Fig.2, a start position of swing leg r_{0sw} and $f_{0sw}(t)$ are as follows.

$$\begin{cases} r_{0sw} = \frac{r_{sw}}{\cos 0sw} & \therefore r_{0sw} = \frac{\delta r_{sw}}{\cos\theta_{0sw}-1} \\ f_{0sw}(t) = -m_{sw}g\cos\theta_{sw}(t) & \end{cases} \qquad (19)$$

By using Eq.(19) and equations are similar to Eq.(3) and Eq.(4), Eq.(17) and Eq.(18) are linearized as follows.

$$r_{0sw}^2\delta\ddot{\theta}_{sw}(t) = \frac{\tau_{sw}(t)}{m_{sw}} - gr_{0sw}\delta\theta_{sw}(t)$$
$$-\frac{\cos\theta_{0sw}}{\cos\theta_{0sw}-1}g\theta_{0sw}\delta r_{sw} \qquad (20)$$
$$\delta\ddot{r}_{sw}(t) = \frac{\delta f_{sw}(t)}{m_{sw}} \qquad (21)$$

Afterwad, swing leg is handled as described in (2.1.2)-(2.1.7). After that, in swing leg, the upper and lower bound of r_{0sw} composed of matrix A_{esw} and B_{esw} are as follows.

$$r_{0sw} \in [r_{0swmin}, \ r_{0swmax}] \qquad (22)$$

Let $\tilde{D}_{d3}$ and $\tilde{D}_{d4}$ be the vertex matrices for the variation range of matrix $\tilde{D}_{dsw}$.

$$\tilde{D}_{d3} = \tilde{D}_{dsw}(r_{0swmin}), \ \tilde{D}_{d4} = \tilde{D}_{dsw}(r_{0swmax}) \ (23)$$

It is assumed that the system stabilizes in the range of the convex hull of the two vertexes.

3 Control Design

I design a controller that guarantees stability robustness for varying parameters r_{0su} and r_{0sw} with the initial state.

3.1 LQ control

To derive a stabilizing state feedback $u(t) = \tilde{K}_d x(t)$, I consider minimizing the following quadratic cost function.

$$J = \int_0^\infty \{\tilde{x_d}(t)^T Q\tilde{x_d}(t) + u(t)^T Ru(t)\}dt \qquad (24)$$

Here $Q \geq 0$ is a weight matrix for state variables, and $R \geq 0$ is a weight matrix for inputs. This is the same quadratic cost function LQ control.

3.2 LMI condition

The following LMI condition is applied for both of support leg and swing leg. The system is stabilized by $u(t) = \tilde{K}_d\tilde{x_d}(t)$ and $J < \gamma$ is guaranteed. In view of structure of $\tilde{E}_d$, we restrict $\tilde{X}_d$, $\tilde{Y}_d$ as follows.

$$\tilde{X}_d = \begin{bmatrix} X_{11} & 0 \\ X_{21} & X_{22} \end{bmatrix}, \ \tilde{Y}_d = [\ Y_1 \quad 0\ ] \qquad (25)$$

Lemma 1 : *If there exists* $X_{11} \succ 0$, $\tilde{X}_d$, $\tilde{Y}_d$ *satisfying the following LMI, the system is stable.*
minimize :γ

subject to $X_{11} \succ 0$

$$\begin{bmatrix} He[\tilde{D}_{dk}\tilde{X}_d + \tilde{B}_d\tilde{Y}_d] & \tilde{X}_d^T(Q^{\frac{1}{2}})^T & \tilde{Y}_d^T(R^{\frac{1}{2}})^T \\ Q^{\frac{1}{2}}\tilde{X}_d & -I & 0 \\ R^{\frac{1}{2}}\tilde{Y}_d & 0 & -I \end{bmatrix} \prec 0 \quad (26)$$

$$(k = 1, 2, 3, 4)$$

$$\begin{bmatrix} \mathrm{W} & I \\ I & X_{11} \end{bmatrix} \succ 0 \quad (27)$$

$$trace(W) \prec \gamma \quad (28)$$

The controller $\tilde{K}_d$ with framework of the descriptor representation is given as follow.

$$\tilde{K}_d = [YX_{11}^{-1} \quad 0] \quad (29)$$

3.3 Switching foots

In this study, by using the following algorithm, support leg and swing leg are switched. 1.Support leg is inclined from start angle to reference angle. 2.Support leg is extended until reference gravity position. 3.Swing leg is swung downward from present angle to ground. 4.Support leg and swing leg are switched.

4 Simulation

4.1 Support leg

To stable the support leg system for variable range, it is important to guarantee the robustness of the upper and lower bound of r_{0su}. And the range of r_{0su} is as follows.

$$r_{0su} \in [r_{0sumin} \; r_{0sumax}] = [0.5, \; 1.0] \quad (30)$$

In this study, let reference angle be 15[deg]. The simulation results of support leg is shown in Fig.3 and Fig.4.

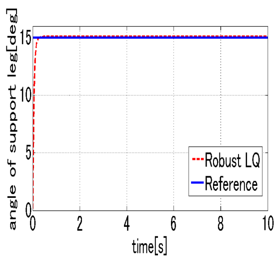

Fig. 3 Angle of support leg

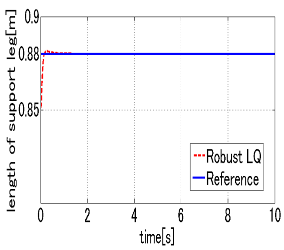

Fig. 4 Length of support leg

4.2 Swing leg

The initial position of swing leg is not defined same position because of swing leg is swung up. So r_{0sw} is a varying parameter. So to stable the swing leg system, it is important to guarantee the robustness of the upper and lower bound of r_{0sw}. The range of r_{0sw} is as follows.

$$r_{0sw} \in [r_{0swmin}, \; r_{0swmax}] = [0.5, \; 1.0] \quad (31)$$

Swing leg is only swing downward so that let reference angle be 0[deg]. The simulation results of swing leg is shown in Fig5 and Fig6.

4.3 Biped robot

In this study, biped robot walking system is composed to support leg and swing leg. The simulation results of biped robot is shown as follows. Fig.7 indicates angle of support leg and swing leg during it walks. Fig.8 indicates length of support leg and swing leg during it walks. Fig.9 indicates center of gravity(COG) of biped robot during it walks.

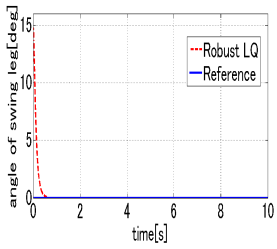

Fig. 5 Angle of swing leg

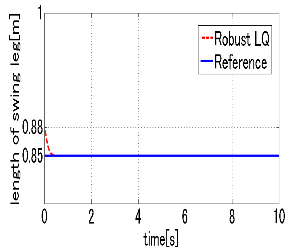

Fig. 6 Length of swing leg

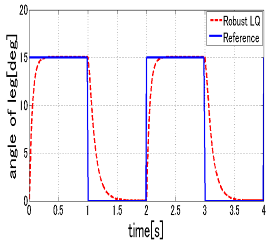

Fig. 7 Angle of support leg and swing leg

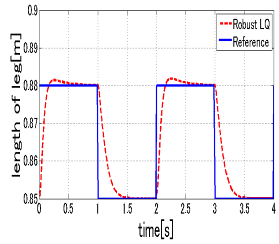

Fig. 8 Length of support leg and swing leg

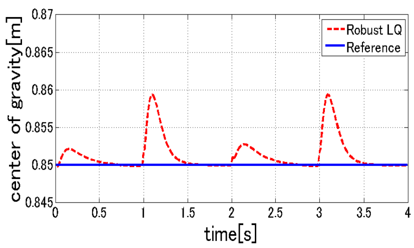

Fig. 9 COG of biped robot

5 Conclusion

In this study, the proposed method to guarantee the robust stability for Biped robot system is shown. The controller is designed to minimize the quadratic cost function. Descriptor representation and LFT are adopted to obtain the system which has multi affine for length of leg. Then, polytopic representation for the each system with these parameters is provided. From results of simulations, the validity of the proposed method is indicated and robust stability is verified. Hence, biped robot walking is achieved.

References

[1] Qiang Huang, Kazuhito Yokoi, Shuuji Kajita, Kenji Kaneko, Hirohiko Arai, Noriho Koyachi and Kazuo Tanie, "Planning Walking Patterns for a Biped Robot", IEEE TRANSACTIONS ON ROBOTICS AND AUTOMATION,VOL.17,NO.3, 2001.

[2] Kemalettin Erbatur and Okan Kurt,"Natural ZMP Trajectories for Biped Robot Reference Generation",IEEE TRANSACTIONS ON INDUSTRIAL ELECTRONICS, VOL. 56, NO. 3, 2009.