Gain Scheduled H_2 Control for Magnetic Bearing with Imbalance

M2014SC012 Tatsuya MIZUNO
Supervisor Isao TAKAMI

Abstract

In this paper, gain scheduled H_2 controller for active magnetic bearing (AMB) system is designed. A rotor has gyroscopic effect, imbalance and resonance. The gyroscopic effect and imbalance depend on angular velocity. The robust stability for angular velocity is guaranteed by using polytopic representation. Descriptor representation and linear fractional transformation (LFT) are adopted to obtain an equivalent polytopic representation of AMB system which has multi-affine. Frequency weight is introduced to suppress vibration by resonance. The problem is formulated as solving a finite set of linear matrix inequalities (LMIs). The effectiveness of the proposed method is illustrated by simulation.

1 Introduction

Active magnetic bearing (AMB) system levitates rotor with contact-free by electromagnetic force. It is possible for AMB to rotate at high rotation velocity. However, when the rotor rotates, the gyroscopic effect is generated. The rotor is vibrated by precession caused by it. Since it depends on the angular velocity of the rotor, as the angular velocity of the rotor increases, the vibration becomes large. Gain scheduled (GS) controller[1] and GS controller via parameter depending Lyapunov function[2] for gyroscopic effect have been reported.

If there exists imbalance of the rotor, the rotor vibrates. The imbalances are two types, which are static imbalance and couple imbalance. The static imbalance occurs because the center of gravity shifts from the center of the rotor. The couple imbalance occurs because inertial axis shifts from rotation axis. The imbalances also depend on the angular velocity and are represented as periodic disturbances[3]. Therefore, the control for AMB is required to consider the vibration caused by not only the gyroscopic effect but also these imbalances. Some literatures for AMB with imbalance have been reported. The H_∞ disturbance and an initial state uncertainty attenuation (DIA) controller for imbalance regarded as frequency disturbance has been reported[4]. The GS sliding mode controller for the static imbalance has been designed[5]. 2-DOF controller based on MIMO decoupling technique, adaptive feedforward algorithm and Notch filter has been designed[6]. The rotor has resonance. This vibrates the rotor when natural frequency is the same as the rotation velocity of the rotor. Some studies for the resonance using flexible rotor model have been reported[7].

In this study, GS H_2 controller via parameter dependent Lyapunov function for AMB is designed. The rotor has the gyroscopic effect, the static and the couple imbalance and the resonance. The object of this study is to suppress vibrations caused by them. The frequency weight is introduced to suppress the vibration caused by the resonance. The dynamics of AMB dependents on angular velocity of the rotor. The robust stability for this parameter is guaranteed theoretically by using polytopic representation. Descriptor representation and linear fractional transformation (LFT) are adopted to obtain an equivalent polytopic representation of AMB system which has the only first-order terms of the varying parameter. The problem is formulated as solving a finite set of linear matrix inequalities (LMIs). The effectiveness of the proposed method is illustrated by simulation comparing with LQ controller.

2 Modeling

2.1 Motion equations

An experimental device used in this study is a 4-axis controlled type AMB with symmetrical structure. Four pairs of electromagnets are located in the horizontal and the vertical of both ends of the rotor. Gap sensors are also located in the horizontal and the vertical of both ends of the rotor. They can measure distance between the electromagnet and the rotor. Coordinates X, Y and Z are introduced to obtain equations of motion as shown Fig. 1. Let p[rad/s], f_j[N] and g_j[m] be rotational velocity of the rotor, levitation force of the electromagnets and displacement from the equilibrium point of the rotor, respectively. Here, subscript $j \in \{lv, rv, lh, rh\}$.

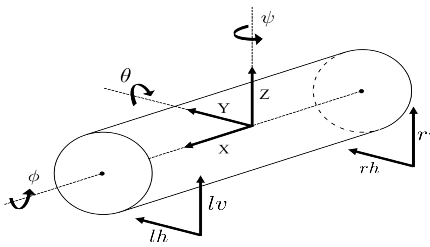

Fig. 1 Coordinates X, Y and Z

These subscripts l, r, v, h mean the left-hand side, right-hand side, vertical direction and horizontal direction, respectively. The imbalances of this study are the static imbalance and the couple imbalance. Here, ε and τ are distance between the center of gravity and the center of the rotor and angle between the inertial axis and the rotation axis. Physical parameters of AMB are shown in Table. 1. The following assumptions are assumed to

Table 1 Physical parameters

Parameter	Symbol	Unit
Mass of rotor	m	[kg]
Acceleration of gravity	g	[m/s^2]
Distance between center of gravity and the left-had side of rotor	l_{ml}	[m]
Distance between center of gravity and the left-had side of rotor	l_{mr}	[m]
Distance between rotor and sensor in the equilibrium state	g_0	[m]
Moment of X axis	J_x	[Nm]
Moment of Y axis	J_y	[Nm]
Suction force coefficient	k	
Constant current of vertical direction	I_{lv}, I_{rv}	[A]
Constant current of horizontal direction	I_{hv}, I_{rh}	[A]

derive the equations of motion.

- The rotor is a rigid body.
- All electromagnets have the same electrical characteristic.
- The center of the gravity shifts in a radical direction from the center of the rotor.

Equations of the translational motion of the directions Y and Z and equations of the rotational motion of the axes Y and X are given by Eq.(1)-(4).

$$m\ddot{y} = f_{lh} + f_{rh} + m\varepsilon p^2 \cos(pt+\kappa) \quad (1)$$

$$m\ddot{z} = -mg + f_{lv} + f_{rv} + m\varepsilon p^2 \sin(pt+\kappa) \quad (2)$$

$$\begin{aligned} J_y\ddot{\theta} &= J_x p\dot{\psi} + l_{ml} f_{lv} - l_{mr} f_{rv} \\ &\quad +(J_y - J_x)\tau p^2 \sin(pt+\lambda) \end{aligned} \quad (3)$$

$$\begin{aligned} J_y\ddot{\psi} &= -J_x p\dot{\theta} - l_{ml} f_{lh} + l_{mr} f_{rh} \\ &\quad +(J_y - J_x)\tau p^2 \cos(pt+\lambda) \end{aligned} \quad (4)$$

Here, y and z are displacement of the directions Y and Z. θ and ψ are rotation angle around the axes Y and Z. κ and λ are the initial values of the phase. The displacement y and z and the rotation angle θ and ψ are represented as Eq.(5)-(6) by using g_j.

$$y = g_{lh} + \frac{(g_{rh} - g_{lh})l_{ml}}{l_{ml} + l_{mr}}, z = g_{lv} + \frac{(g_{rv} - g_{lv})l_{mr}}{l_{ml} + l_{mr}} \quad (5)$$

$$\theta \approx \frac{g_{lv} - g_{rv}}{l_{ml} + l_{mr}}, \psi \approx \frac{g_{rh} - g_{lh}}{l_{ml} + l_{mr}} \quad (6)$$

The levitation force of the electromagnetic is given by Eq.(7)

$$f_j = k\frac{(I_0 + I_j + i_j)^2}{(g_j - g_0)^2} - k\frac{(I_0 - I_j - i_j)^2}{(g_j + g_0)^2} \quad (7)$$

Here, I_0, I_j and i_j are bias current, steady-state current and control input, respectively. Since the position of the rotor does not change significantly, Eq.(7) is linearized at the equilibrium point as follows.

$$f_j = k\frac{4I_0 I_j}{g_0^2} + K_{xj} g_j + K_{ij} i_j \quad (8)$$

$$K_{xj} = k\frac{4(I_0^2 + I_j^2)}{g_0^3}, K_{ij} = k\frac{4I_0}{g_0^2} \quad (9)$$

Eq.(10)-(13) are obtained by Eq.(1)-(9).

$$\begin{aligned} \ddot{g_{lv}} &= aK_{xlv} g_{lv} + cK_{xrv} g_{rv} - pdl_{ml}\dot{g_{lh}} + pdl_{ml}\dot{g_{rh}} \\ &\quad + aK_{ilv} i_{lv} + cK_{irv} i_{rv} + p^2\alpha_1 + p^2 l_{ml}\beta_1 \quad (10) \\ \ddot{g_{rv}} &= cK_{xlv} g_{lv} + bK_{xrv} g_{rv} - pdl_{mr}\dot{g_{lh}} + pdl_{mr}\dot{g_{rh}} \\ &\quad + cK_{ilv} i_{lv} + bK_{irv} i_{rv} + \alpha_1 p^2 - p^2 l_{mr}\beta_1 \quad (11) \\ \ddot{g_{lh}} &= aK_{xlh} g_{lh} + cK_{xrh} g_{rh} + pdl_{ml}\dot{g_{lv}} - pdl_{ml}\dot{g_{rv}} \\ &\quad + aK_{ilh} i_{lh} + cK_{irh} i_{rh} + p^2\alpha_2 - p^2 l_{ml}\beta_2 \quad (12) \\ \ddot{g_{rh}} &= cK_{xlh} g_{lh} + bK_{xrh} g_{rh} - pdl_{mr}\dot{g_{lv}} + pdl_{mr}\dot{g_{rv}} \\ &\quad + cK_{ilh} i_{lh} + bK_{irh} i_{rh} + p^2\alpha_2 + p^2 l_{mr}\beta_2 \quad (13) \end{aligned}$$

$$a = \frac{1}{m} + \frac{l_{ml}^2}{J_y}, b = \frac{1}{m} + \frac{l_{mr}^2}{J_y}, c = \frac{1}{m} - \frac{l_{ml} l_{mr}}{J_y}$$

$$d = \frac{J_x}{J_y(l_{ml} + l_{mr})}$$

$$\alpha_1 = \varepsilon \sin(pt+\kappa), \beta_1 = (1 - \frac{J_x}{J_y})\tau \sin(pt+\lambda)$$

$$\alpha_2 = \varepsilon \cos(pt+\kappa), \beta_2 = (1 - \frac{J_x}{J_y})\tau \cos(pt+\lambda)$$

2.2 State space representation

From Eq.(10)-(13), state space representation is obtained by as follows.

$$\begin{cases} \dot{x}(t) = Ax(t) + B_1 w(t) + B_2 u(t) \\ y(t) = C_2 x(t) \end{cases} \quad (14)$$

$$x(t) = [g_{lv}\ g_{rv}\ g_{lh}\ g_{rh}\ \dot{g_{lv}}\ \dot{g_{lv}}\ \dot{g_{lv}}\ \dot{g_{lv}}]^T$$

$$u(t) = [i_{lv} i_{rv}\ i_{lh}\ i_{rh}]^T$$

$$w(t) = \begin{bmatrix} \varepsilon \sin(pt+\kappa) \\ \tau \sin(pt+\lambda) \\ \varepsilon \cos(pt+\kappa) \\ \tau \cos(pt+\lambda) \end{bmatrix}, A = \begin{bmatrix} O & I \\ A_1 & pA_2 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} O \\ p^2 B_{11} \end{bmatrix}, B_2 = \begin{bmatrix} O \\ B_{21} \end{bmatrix}, C = I_{8\times 8}$$

Here, matrices A_1, A_2, B_{11} and B_{21} are constant matrices obtained by Eq.(10)-(13).

3 Frequency weight

The frequency weight that has the peak of a gain at the natural frequency is introduced for the state variable to suppress the vibration that is caused by the resonance. A frequency weight function for the state variable $W_f(S)$ is given as follow.

$$W_f(s) = I_{8\times 8} W(s), W(s) = W_C(Is - W_A)^{-1} W_B \quad (15)$$

The frequency weight function $W_f(S)$ is represented as the state space representation (16).

$$\begin{aligned} \dot{x}_w(t) &= A_w x_w(t) + B_w x(t) \\ \bar{z}(t) &= C_w x_w(t) \end{aligned} \quad (16)$$

Here, $x_w(t)$ is the state of the frequency weight function $W_f(s)$. Let $x_f(t)$ be a new state variable $x_f(t) = [x(t)^T\ \ x_w(t)^T]^T$. Then generalized plant for a extended system including the frequency weight is as follows.

$$\begin{cases} \dot{x}_f(t) = A_f(p)x_f(t) + B_{f1}(p^2)w(t) + B_{f2}u(t) \\ z_f(t) = C_{f1}x_f(t) + D_{f12}u(t) \end{cases} \quad (17)$$

$$A_f = \begin{bmatrix} A & O \\ B_w & A_w \end{bmatrix}, B_{f1} = \begin{bmatrix} B_1 \\ O \end{bmatrix}, B_{f2} = \begin{bmatrix} B_2 \\ O \end{bmatrix}$$

$$C_{f1} = \begin{bmatrix} W_x & O \\ O & C_w \\ O & O \end{bmatrix}, D_{f12} = \begin{bmatrix} O \\ O \\ W_u \end{bmatrix}$$

Here, z_f, $W_x \succ 0$ and $W_u \succ 0$ are a evaluated output, weight matrices for the state variable and the input.

4 Transformation into multi-affine

Since the state space representation of the plant model has the first-order and second-order terms of varying parameter, it is difficult to use polytopic representation directly. The system is transformed to an equivalent system that is multi-affine for p by using descriptor representation and linear fractional transformation (LFT).

4.1 Descriptor representation

Matrix B_{f1} of Eq.(17) has the second-order terms of the angular velocity of the rotor. All varying parameter are put into one matrix by expanding dimension of matrices to apply LFT. The varying parameter p is put into matrix A_{fd} by defining $x_d(t) = [x(t) \quad w(t)]^T$.

$$E_{fd}\dot{x}_{fd}(t) = A_{fd}(p^2)x_{fd}(t) + B_{fd1}w(t) + B_{fd2}u(t) \quad (18)$$

$$E_{fd} = \begin{bmatrix} I & O \\ O & O \end{bmatrix}, A_{fd}(p) = \begin{bmatrix} A_f(p) & B_{f1}(p^2) \\ O & -I \end{bmatrix}$$

$$B_{fd1} = \begin{bmatrix} O \\ I \end{bmatrix}, B_{fd2} = \begin{bmatrix} B_{f2} \\ O \end{bmatrix}$$

4.2 Linear fractional transformation

LFT is adapted to eliminate the products of p. Matrix A_{fd} can be represented as Eq.(19). Here A_{fdn} is a constant matrix in A_{fd}, and $B_\delta \Delta C_\delta$ is a matrix which contains the first and second-order terms of p in A_{fd} by choosing appropriate A_{fdn}, B_δ, C_δ and Δ as follows.

$$A_{fd}(p) = A_{fdn} + B_\delta \Delta(p) C_\delta(p) \quad (19)$$

$$A_{fdn} = \begin{bmatrix} A_0 & O & O \\ B_w & A_w & O \\ O & O & -I \end{bmatrix}, B_\delta = \begin{bmatrix} B_{\delta 0} \\ O \\ O \end{bmatrix}$$

$$C_\delta(p) = [\ A_1 \quad O \quad pB_{11}\ ], B_{\delta 0} = \begin{bmatrix} I \\ O \end{bmatrix}$$

$$\Delta(p) = diag(p \quad p \quad p \quad p)$$

Note that C_δ have first-order terms of p. A_{fdn} and B_δ is constant matrix. Let $\tilde{x}_{fd}$ be the new descriptor variable $\tilde{x}_d = [x_d^T \quad z_\delta^T]^T$. Descriptor equation is obtained as follows.

$$\tilde{E}_{fd}\dot{\tilde{x}}_{fd}(t) = \tilde{A}_{fd}(p)\tilde{x}_{fd}(t) + \tilde{B}_{fd1}w(t) + \tilde{B}_{fd2}u(t) \quad (20)$$

$$\tilde{E}_{fd} = \begin{bmatrix} E_{fd} & O \\ O & O \end{bmatrix}, \tilde{A}_{fd}(p) = \begin{bmatrix} A_{fdn} & B_\delta \Delta(p) \\ C_\delta(p) & -I \end{bmatrix}$$

$$\tilde{B}_{fd1} = \begin{bmatrix} B_{fd1} \\ O \end{bmatrix}, \tilde{B}_{fd2} = \begin{bmatrix} B_{fd2} \\ O \end{bmatrix}, z_\delta = C_\delta x_{fd}(t)$$

5 Control design

In this section, the GS controller via parameter dependent Lyapunov function is synthesized. The scheduling parameter is angular velocity of the rotor p. The stability of the closed system should be guaranteed theoretically for variation of the angular velocity of the rotor. The robust stability for the time varying parameter p is guaranteed by using polytopic representation. The range of the time varying parameter p is defined by upper and lower bound. The objective of this study is to guarantee the robustness in the following range.

$$p \in [\underline{p}, \overline{p}] = [p_1, p_2] \quad (21)$$

From Eq.(21), matrix $\tilde{A}_{fd}$ is represented by polytopic representation Eq.(22)

$$\tilde{A}_{fd} = \alpha \tilde{A}_{fd}(p_1) + (1-\alpha)\tilde{A}_{fd}(p_2), \alpha \in [0,1] \quad (22)$$

Eq.(22) shows that $\tilde{A}_{fd}(p) = \tilde{A}_{fd}(p_1)$ when p is minimum and $\tilde{A}_{fd}(p) = \tilde{A}_{fd}(p_1)$ when p is maximum. For Eq.(20), H_2 norm from the disturbance $w(t)$ to the evaluated output $z(t)$ is given as Eq.(23)

$$||G||_2^2 = \int_0^\infty z(t)^T z(t) dt \quad (23)$$

In order to minimize the H_2 norm, minimum γ that satisfy the following conditions are derived. Here, state feedback controller $u = \tilde{K}_{fd}(p)\tilde{x}_{fd}$ and feedback gain $\tilde{K}_{fd}(p) = \tilde{Y}_{fd}(p)\tilde{X}_{fd}(p)^{-1}$.

$$He[\tilde{A}_{fd}(p)\tilde{X}_{fd}(p) + \tilde{B}_{fd2}\tilde{Y}_{fd}(p)] - \tilde{E}_{fd}\dot{\tilde{X}}_{fd}(p) + \tilde{B}_{fd1}\tilde{B}_{fd1}^T \prec 0 \quad (24)$$

$$\tilde{E}_{fd}\dot{\tilde{X}}_{fd}(p) = (\tilde{E}_{fd}\dot{\tilde{X}}_{fd}(p))^T \succ 0 \quad (25)$$

$$C_{f1}X_f(p)C_{f1}^T \prec Z \quad (26)$$

$$trace(Z) < \gamma^2 \quad (27)$$

In view of structure of the matrix $\tilde{E}_{fd}$, candidates of Lyapunov matrix $\tilde{X}_{fd}(p)$, variable matrix $\tilde{Y}_{fd}(p)$ and $\tilde{E}_d\dot{\tilde{X}}_{fd}(p)$ are restricted as follow.

$$\tilde{X}_{fd}(p) = \begin{bmatrix} X_f(p) & O & O \\ X_{21}(p) & X_{22}(p) & X_{23}(p) \\ X_{31}(p) & X_{32}(p) & X_{33}(p) \end{bmatrix} \quad (28)$$

$$\tilde{Y}_{fd}(p) = [\ Y_f(p) \quad O \quad O\ ] \quad (29)$$

$$\tilde{E}_{fd}\dot{\tilde{X}}_{fd}(p) = \begin{bmatrix} X_f(\dot{p}) - X_{f0} & O & O \\ O & O & O \\ O & O & O \end{bmatrix} \quad (30)$$

In Eq.(24), there exists product of scheduling parameter $\tilde{A}_{fd}(p)\tilde{X}_{fd}(p)$. $\tilde{X}_{fd}(p)$ is restricted as Eq.(31). Then Eq.(24) becomes multi-affine for varying parameter. $X_f(p)$, $\tilde{Y}_{fd}(p)$ and $Y_f(p)$ are also assigned as follows.

$$\tilde{X}_{fd}(p) = \tilde{X}_{fd0} + p\tilde{X}_{fd1}, \tilde{X}_{fd1} = \begin{bmatrix} X_{f1} & O & O \\ O & O & O \\ O & O & O \end{bmatrix} \quad (31)$$

$$X_f(p) = X_{f0} + pX_{f1} \quad (32)$$

$$\tilde{Y}_{fd}(p) = \tilde{Y}_{fd0} + p\tilde{Y}_{fd1}, Y_f(p) = Y_0 + pY_1 \quad (33)$$

LMI conditions that minimize H_2 norm and stabilize the system are as follows.

Lemma 1 : *If there exist $\tilde{X}_{fd}(p)$ and $\tilde{Y}_{fd}(p)$ satisfying this LMIs, the system is stabilized.*
minimize :γ^2
subject to

$$X_f(p_i) \succ 0 \quad (34)$$

$$\begin{bmatrix} M(p_i) - \tilde{E}_{fd}\dot{\tilde{X}}_{fd}(p_i) & \tilde{B}_{fdw} \\ \tilde{B}_{fdw}^T & -I \end{bmatrix} \prec 0 \quad (35)$$

$$\begin{bmatrix} Z & (C_{f1}X_f(p_i) + D_{f12}Y_f(p_i)) \\ (C_{f1}X_f(p_i) + D_{f12}Y_f(p_i))^T & X_f(p_i) \end{bmatrix} \succ 0 \quad (36)$$

$$trace(Z) < \gamma^2 \quad (37)$$

$$M(p_i) := He[\tilde{A}_{fd}(p_i)\tilde{X}_{fd}(p_i) + \tilde{B}_{fd2}\tilde{Y}_{fd}(p_i)], (i = 1, 2)$$

Gain scheduled controller $K_f(p) = Y_f(p)X_f(p)^{-1}$ with framework of the state space representation is obtained.

6 Simulation

In this section, the validity of the proposed method is illustrated by comparing with LQ controller in simulation. In this study, the range of the angular velocity is assumed from 0[rad/s] to 2618[rad/s](25,000[rpm]). The rotor rotates as Fig. 2. The distance between the center and the center of gravity of rotor $\varepsilon = 1.0 \times 10^{-6}$[m] and the angle of rotation axis to inertia axis $\tau = 1.75 \times 10^{-5}$[rad]. The initial value of state of the rotor is $x(0) = [-1.5 \times 10^{-6} \ 1.5 \times 10^{-6} \ -1.0 \times 10^{-6} \ 1.0 \times 10^{-6} \ 0 \ 0 \ 0 \ 0]$. LQ controller is designed at 25,000[rpm]. The frequency weight function $W(s)$ is as follow.

$$W(s) = \frac{100.05}{s^2 + 2.64 \times 10^{-2}s + 1.75 \times 10^6} \quad (38)$$

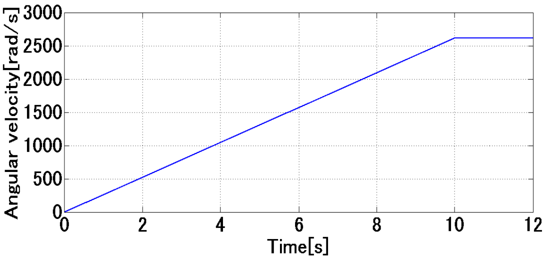

Fig. 2 Angular velocity

Fig. 3 and 4 show the force and the torque that are caused by the static and the couple imbalance. The simulation results of the displacements from the equilibrium point on the vertical and horizontal direction of the left hand side are shown in Fig. 5 and 6. The convergence of the rotor controlled by H_2 controller is later than LQ controller. However, H_2 controller suppresses the vibration that is caused the static and the couple imbalance than LQ controller.Since the rotor is more suppressed at about 5 seconds, the effect of the frequency weight is shown. The simulation results of the input current on the vertical direction of the left hand side and the right hand side are shown in Fig. 7 and 8. The simulation result of torques that are caused by the gyroscopic effect are shown in Fig. 9 and 10. The torques that are caused by the gyroscopic effect are also suppressed by H_2 controller than LQ controller.

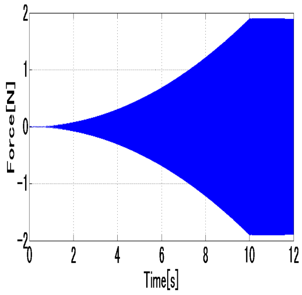

Fig. 3 Force cased by static imbalance

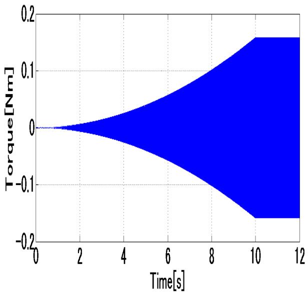

Fig. 4 Torque caused by couple imbalance

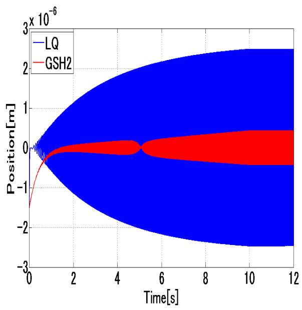

Fig. 5 Displacement g_{l1}

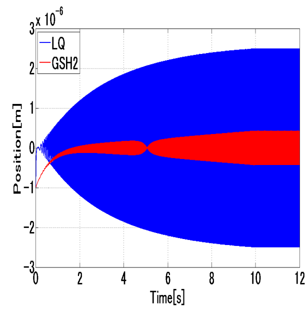

Fig. 6 Displacement g_{l3}

7 Conclusion

In this paper, the gain scheduled H_2 controller for the active magnetic bearing (AMB) is designed. The rotor has the gyroscopic effect, the imbalances and the resonance. This controller is synthesized with parameter dependent Lyapunov function. The robust stability for the angular velocity is guaranteed theoretically by using polytopic representation. Descriptor representation and linear fractional transformation (LFT) are adopted to

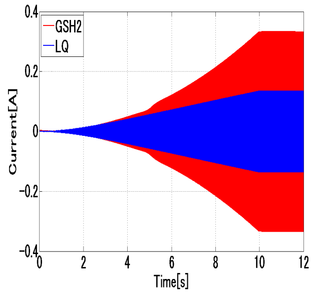

Fig. 7 Control input i_{l1}

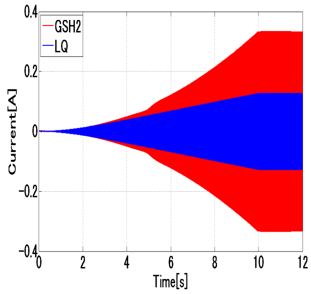

Fig. 8 Control input i_{l3}

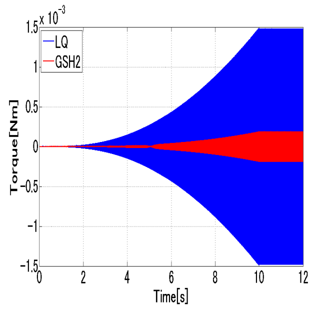

Fig. 9 Torque caused by gyroscopic effect of Y

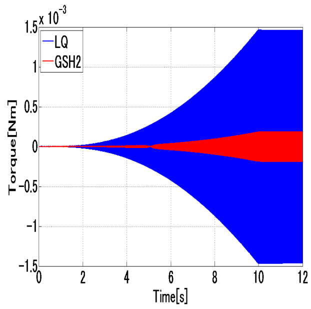

Fig. 10 Torque caused by gyroscopic effect of Z

obtain an equivalent polytopic representation of AMB system which has multi-affine. The frequency weight is introduced to suppress the vibration caused by the resonance. The problem is formulated as solving a finite set of linear matrix inequalities (LMIs). The effectiveness of the proposed method is illustrated by simulation by comparing with LQ controller. The proposed method suppresses the vibration of the rotor than LQ controller and suppresses more at the natural frequency by the frequency weight.

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