Nonlinear Tracking Control for Control Moment Gyroscope with Nonholonomic Constraint

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Abstract

This research describes the design of tracking controller with integrator to compensate friction for Control Moment Gyroscope which is a first-order nonholonomic system. Backstepping method is used to design the controller for the chained system. Stability of the system is guaranteed theoretically based on Lyapunov function. The effectiveness of the controller is shown by simulations and experiments.

1 Introduction

In this paper, a control strategy for a class of nonholonomic systems is proposed. It is proven than nonholonomic systems cannot be stabilized by using linear time-invariant state feedback even if the controllability is ensured in the sense of nonlinear system [1]. The chained form is applied as the canonical one to the first-order nonholonomic system [2].

Control Moment Gyro (CMG), which is treated in this research, is also a first-order nonholonomic system. CMGs are applied to attitude control of large scale spacecraft. They provide huge torque by tilting a gimbal, which contains spinning rotor. CMGs are generally used in collaboration. In this research, control for single CMG is discussed as a fundamental research for CMGs. Recently efficient methods for single CMG are proposed, for example tracking controller using backstepping approach [3], optimal controller based on center-stable manifold computation [4], and tracking controller with an integrator [5].

There exists friction in CMG which cause the difficulty to control. This paper describes the design of a tracking controller which makes the rotating bodies track the reference trajectory without error despite the friction of the system. The contribution of this research is the proposal that the rotor can provide torques to move the gimbals while keeping stability despite the friction of the system. The tracking controller is designed for the chained form system based on backstepping approach. The influence of friction is dissolved by the controller with integrator. Stability of the system is guaranteed theoretically based on Lyapunov function. The effectiveness of the controller is illustrated by simulations and experiments.

2 Mathematical Model

The schematic model of CMG is shown in Figure 1. CMG consists of four rigid bodies, rotor1, gimbal2, gimbal3 and gimbal4, giving four angular degree of freedom. Note that gimbal3 and gimbal4 do not have any drive sources. Gimbal3 is locked in this research. The angle q_1 and the angular velocity ω_1 are defied as the rotation about a_2 of rotor1 relative to gimbal2. The angle q_2 and the angular velocity ω_2 are defined as the rotation about b_1 of gimbal2 relative to gimabl3. Gimbal2 has hardware restriction on its motion range and the singularity. Thus the trajectory of gimbal2 is discussed under $0 < q_2 < (\pi/2)$ in this research. The angle q_4 and the angular velocity ω_4 are defined as the rotation about c_3

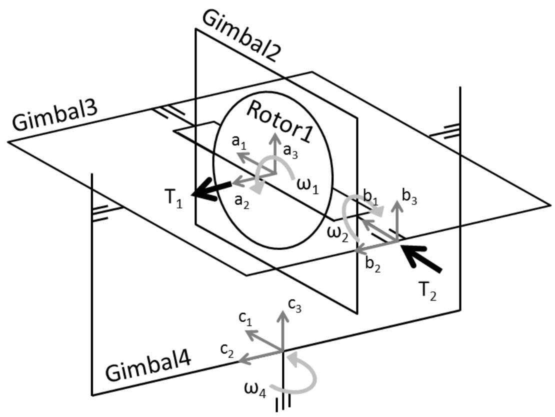

Figure 1 Schematic Model of CMG

of gimbal4. An input of motor1 attached in rotor1 is T_1. Another input of motor2 attached in gimbal2 is T_2.

2.1 Nonlinear Dynamics

The equation of motions of each bodies, rotor1, gimbal2 and gimbal4, are obtained as eq.(1)-(3) where I_i is the moment of inertia. The friction is F_i.

$$I_{R1y}\dot{\omega}_1 + I_{R1y}\dot{\omega}_4 \sin q_2 + I_{R1y}\omega_2\omega_4 \cos q_2 = T_1 + F_1 \quad (1)$$

$$\begin{aligned}(I_{G2x} + I_{R1x})\dot{\omega}_2 - I_{R1y}\omega_1\omega_4 \cos q_2 \\ - I_1\omega_4^2 \sin q_2 \cos q_2 = T_2 + F_2\end{aligned} \quad (2)$$

$$\begin{aligned}I_{R1y}\dot{\omega}_1 \sin q_2 + (I_2 + I_1 \sin^2 q_2)\dot{\omega}_4 \\ + I_1\omega_2\omega_4 \sin 2q_2 + I_{R1y}\omega_1\omega_2 \cos q_2 = F_4\end{aligned} \quad (3)$$

The constraint equation is derived as eq.(4) by the integral of eq.(3) when the initial angular momentums of each bodies are zero.

$$I_{R1y}\omega_1 \sin q_2 + (I_2 + I_1 \sin^2 q_2)\omega_4 = 0 \quad (4)$$

2.2 Chained Form

According to the ordinary converting algorithm [6], the constraint equation (4) is converted to the chained form system applying the following conversions.

$$\left\{\begin{array}{l} x_1 = q_1 \\ x_2 = \alpha(q_2) \\ x_3 = q_4 \end{array}\right. , \left\{\begin{array}{l} u_1 = \omega_1 \\ u_2 = \beta(q_2)\omega_2 \end{array}\right. \quad (5)$$

$$\alpha(q_2) = \frac{-I_{R1y}\sin q_2}{I_2 + I_1 \sin^2 q_2},\ \beta(q_2) = \frac{\mathrm{d}}{\mathrm{d}q_2}\alpha(q_2)$$

Then the chained form system is obtained as eq.(6).

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ x_2 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad (6)$$

This system ensures global controllability because a chained form system globally satisfies Lie algebra rank condition.

3 Controller Synthesis

We design the controller that makes gimbal2 track the trajectory under the restriction and the angle of gimbal4 track the reference trajectory without error. Hence the error dynamics is constructed for gimbal2 and gimbal4. In addition, the controller with an integrator is considered for gimbal4 because of compensating the friction.

3.1 Error Dynamics

Let the reference trajectories of gimbal2 and gimbal4 be q_2^{ref} and q_4^{ref}. According to the coordinate and input conversions (5), the reference trajectories in chained form are defined as eq.(7). The tracking error variables are defined as eq.(8).

$$x_2^{\mathrm{ref}} = \alpha(q_2^{\mathrm{ref}}),\ x_3^{\mathrm{ref}} = q_4^{\mathrm{ref}},\ u_2^{\mathrm{ref}} = \beta(q_2^{\mathrm{ref}})\dot{q}_2^{\mathrm{ref}} \quad (7)$$

$$x_{2e} = x_2 - x_2^{\mathrm{ref}},\ x_{3e} = x_3 - x_3^{\mathrm{ref}} \quad (8)$$

Then the state variable of the error dynamics is defined as $x_e = [x_1\ x_{2e}\ x_{3e}\ s_3]$, where s_3 is the integral of the error x_{3e}. The error dynamics is defined as eq.(9).

$$\begin{cases} \dot{x}_1 = u_1 \\ \dot{x}_{2e} = u_2 - u_2^{\mathrm{ref}} \\ \dot{x}_{3e} = x_2 u_1 - \dot{x}_3^{\mathrm{ref}} \\ \dot{s}_3 = x_{3e} \end{cases} \quad (9)$$

3.2 Control Design of Kinematic System

The error dynamics (9) is divided into the following two subsystems (10) and (11).

$$\Delta_1 : \dot{x}_{2e} = u_2 - u_2^{\mathrm{ref}} \quad (10)$$

$$\Delta_2 : \begin{cases} \dot{x}_1 = u_1 \\ \dot{x}_{3e} = x_2 u_1 - \dot{x}_3^{\mathrm{ref}} \\ \dot{s}_3 = x_{3e} \end{cases} \quad (11)$$

The two subsystems are stabilized using backstepping approach. Firstly the feedback controller is applied as eq.(12) to stabilize the subsystem Δ_1.

$$u_2 = u_2^{\mathrm{ref}} - k_1 x_{2e}, \quad k_1 > 0 \quad (12)$$

Secondly the subsystem Δ_2 is stabilized using backstepping controller. The feedback controller is applied as eq.(13) to stabilize s_3 by regarding x_{3e} as a virtual input.

$$x_{3e} = -k_2 s_3, \quad k_2 > 0 \quad (13)$$

Then the error σ between the left side and right side of eq.(13) is defined as eq.(14).

$$\sigma = x_{3e} + k_2 s_3 \quad (14)$$

Consider to stabilize the dynamics of σ because it is necessary for x_{3e} to satisfy eq.(13).

Theorem 1 The dynamics of s_3, σ becomes asymptotically stable if the following equation is satisfied.

$$u_1 = \left\{\dot{x}_3^{\mathrm{ref}} + \left(k_2^2 - 1\right) s_3 - \left(k_2 + k_3\right)\sigma\right\}/x_2, \quad k_3 > 0,\ x_2 \neq 0 \quad (15)$$

Proof If the Lyapunov function candidate is chosen as eq.(16), the time derivative is calculated as negative function (17) by using eq.(15).

$$V_1 = \frac{1}{2}s_3^2 + \frac{1}{2}\sigma^2 > 0 \quad (16)$$

$$\dot{V}_1 = -k_2 s_3^2 - k_3\sigma^2 < 0 \quad (17)$$

3.3 Control Design of Dynamic System

The chained form system inputs u_1 and u_2 are designed as eq.(12) and (15). From the input conversions (5), the error ξ_1 between $\beta(q_2)\omega_2$ and the right hand side of eq.(12), and the error ξ_2 between ω_1 and and the right hand side of eq.(15) are defined.

$$\xi_1 = \beta(q_2)\omega_2 - (u_2^{\mathrm{ref}} - k_1 x_{2e}) \quad (18)$$

$$\xi_2 = \omega_1 - \left\{\dot{x}_3^{\mathrm{ref}} + \left(k_2^2 - 1\right) s_3 - \left(k_2 + k_3\right)\sigma\right\}/x_2 \quad (19)$$

Consider to stabilize the dynamics of ξ_1 and ξ_2 because it is necessary for ω_1 and $\beta(q_2)\omega_2$ to satisfy eq.(15) and (12), respectively.

Theorem 2 The dynamics of x_{2e}, ξ_1 becomes asymptotically stable if the following equation is satisfied.

$$\dot{\omega}_2 = \left\{\dot{u}_2^{\mathrm{ref}} + \left(k_1^2 - 1\right) x_{2e} - \left(k_1 + H_1\right)\xi_1 - \gamma(q_2)\omega_2^2\right\}/\beta(q_2) \quad (20)$$

$$H_1 > 0,\ \gamma(q_2) = \frac{\mathrm{d}\beta(q_2)}{\mathrm{d}q_2}$$

Proof If the Lyapunov function candidate is chosen as eq.(21), the time derivative is calculated as negative function (22) by using eq.(20).

$$V_2 = \frac{1}{2}x_{2e}^2 + \frac{1}{2}\xi_1^2 > 0 \quad (21)$$

$$\dot{V}_2 = -k_1 x_{2e}^2 - H_1\xi_1^2 < 0 \quad (22)$$

Theorem 3 The dynamics of s_3, σ and ξ_2 becomes asymptotically stable if the following equation is satisfied.

$$\dot{\omega}_1 = \left\{G_1 s_3 + (G_2 - x_2^3)\sigma + (G_3 - H_2 x_2^2)\xi_2 + G_4\dot{x}_3^{\mathrm{ref}} + x_2\ddot{x}_3^{\mathrm{ref}}\right\}/x_2^2, \quad H_2 > 0 \quad (23)$$

Where G_i $(i = 1, 2, 3, 4)$ are the functions of x_2 and ξ_1 as follows.

$$G_1 = 2k_2x_2 - k_2^3x_2 + k_3x_2 - k_2^2\xi_1 + k_1k_2^2x_{2e} - k_2^2\dot{x}_2^{\mathrm{ref}} + \xi_1 - k_1x_{2e} + \dot{x}_2^{\mathrm{ref}},$$

$$G_2 = k_2k_3x_2 + k_2^2x_2 - x_2 + k_3^2x_2 + k_2\xi_1 - k_1k_2x_{2e} + k_2\dot{x}_2^{\mathrm{ref}} + k_3\xi_1 - k_1k_3x_{2e} + k_3\dot{x}_2^{\mathrm{ref}},$$

$$G_3 = -k_2x_2^2 - k_3x_2^2,$$

$$G_4 = -\xi_1 + k_1x_{2e} - \dot{x}_2^{\mathrm{ref}},\ x_2 \neq 0$$

Proof If the Lyapunov function candidate is chosen as eq.(24), the time derivative is calculated as negative function (25) by using eq.(23).

$$V_3 = \frac{1}{2}s_3^2 + \frac{1}{2}\sigma^2 + \frac{1}{2}\xi_2^2 > 0 \quad (24)$$

$$\dot{V}_3 = -k_2s_3^2 - k_3\sigma^2 - H_2\xi_2^2 < 0 \quad (25)$$

Then eq.(20) and (23) are substituted for eq.(1)-(3) to derive the dynamic system inputs T_1 and T_2. The inputs are calculated as follows.

$$T_1 = \left\{G_1 s_3 + (G_2 - x_2^3)\sigma + (G_3 - H_2 x_2^2)\xi_2 + G_4\dot{x}_3^{\mathrm{ref}} + x_2\ddot{x}_3^{\mathrm{ref}} - f_1x_2^2\right\}/f_2x_2^2 \quad (26)$$

$$
\begin{aligned}
T_2 =& (I_{G2x}+I_{R1x})\{\dot{u}_2^{\text{ref}} + (k_1^2-1)\,x_{2e} \\
&-(k_1+H_1)\,\xi_1 - \gamma(q_2)\omega_2^2 - f_3\beta(q_2)\}/\beta(q_2) \quad (27)\\
f_1 =& \frac{f_{1a}}{I_2 + I_1\sin^2 q_2 - I_{R1y}\sin^2 q_2}\\
f_{1a} =& I_1\omega_2\omega_4\sin^2 q_2\cos q_2 - I_2\omega_2\omega_4\cos q_2\\
&+I_{R1y}\omega_1\omega_2\sin q_2\cos q_2\\
f_2 =& \frac{I_2 + I_1\sin^2 q_2}{I_{R1y}(I_2+I_1\sin^2 q_2 - I_{R1y}\sin^2 q_2)}\\
f_3 =& \frac{I_{R1y}\omega_1\omega_4\cos q_2 + I_1\omega_4^2\sin q_2\cos q_2}{I_{G2x}+I_{R1x}}\\
& x_2 \neq 0,\ \ \beta(q_2)\neq 0
\end{aligned}
$$

The dynamical system of CMG becomes asymptotically stable by inputs T_1 and T_2. Here note that $q_2 = 0$ is an singularity because the torque becomes infinite if $q_2 = 0$ in eq.(27).

4 Simulation

The effectiveness of this research is illustrated by simulations. Simulations including friction are executed. The friction are measured by some experiments. The equation of friction is defined as eq.(28), where F_{is}, F_{ic} and F_{iv} $(i = 1, 2, 4)$ are the coefficients of static friction, coulomb friction and viscous friction respectively.

$$
F_i = \begin{cases} F_{is} & (\omega_i = 0)\\ F_{ic}\text{sgn}\omega_i + F_{iv}\omega_i & (\omega_i \neq 0)\end{cases} \qquad (i=1,2,4) \quad (28)
$$

Initial conditions are given as $[\ q_1\ q_2\ q_4\ ] = [\ 0\ \frac{\pi}{18}\ 0\ ]$ [rad]. References are given as eq.(29) and (30).

$$
q_2^{\text{ref}}[\text{rad}] = \begin{cases} \frac{\pi}{18} & (t<4)\\ -\frac{7}{36}\pi\sin(\frac{\pi}{4}t-\frac{\pi}{2})+\frac{1}{2} & (4\le t\le 8)\\ \frac{7}{72}\pi\sin(\frac{\pi}{2}t-\frac{3}{2}\pi)+\frac{25}{72}\pi & (t>8)\end{cases} \quad (29)
$$

$$
q_4^{\text{ref}}[\text{rad}] = \begin{cases} 0 & (t<4)\\ -\frac{1}{2}\sin(\frac{\pi}{4}t-\frac{\pi}{2})+\frac{1}{2} & (4\le t\le 8)\\ 1 & (t>8)\end{cases} \quad (30)
$$

Gain parameters are chosen by trial and error as follows.

$$
k_1 = 5, k_2 = 0.01, k_3 = 0.04, H_1 = 10, H_2 = 1.8 \quad (31)
$$

Simulations are shown in Figure 2-6. The solid line shows the proposed method (proposed) and the dashed line shows the controller without integrator (w/o integrator). The dotted line shows the reference. It can be seen that the response of ω_1 is stabilized in Figure 2. The angle of gimbal2 has constraint $0 < q_2 < (\pi/2)$. As can be seen that the response of q_2 tracks the reference under the restriction in Figure 3. The angle of gimbal4, which does not have any drive sources, is shown in Figure 4. The response of q_4 tracks the reference without error by applying proposed method in case that the friction exist in the system. Control torques are shown in Figure 5, 6. Constraints of the torques are $|T_1| < 0.6$ [Nm] and $|T_2| < 2.4$ [Nm]. In the proposed method, the system can be controlled within the constraints.

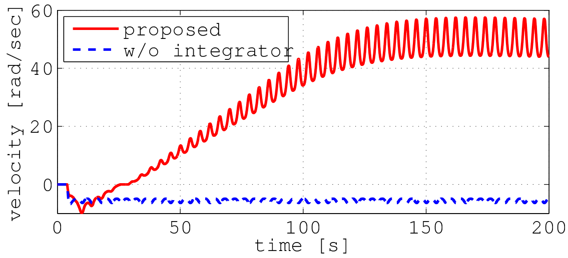

Figure 2 Simulation of ω_1

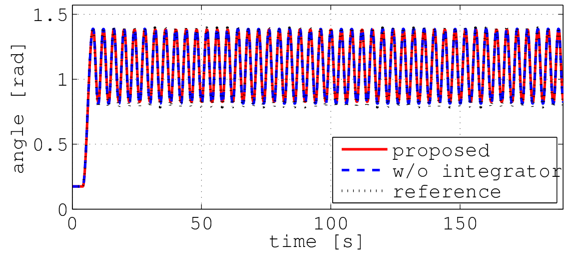

Figure 3 Simulation of q_2

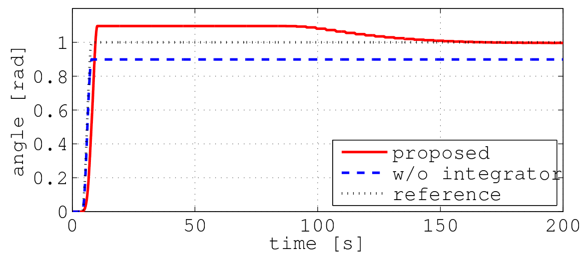

Figure 4 Simulation of q_4

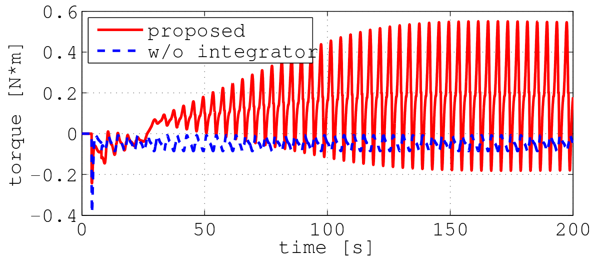

Figure 5 Simulation of torque T_1

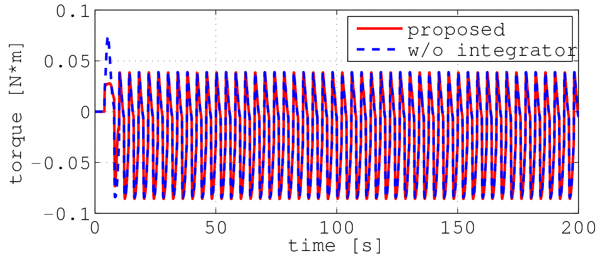

Figure 6 Simulation of torque T_2

5 Experiment

The effectiveness of the proposed controller is confirmed by experiments. The experimental results are shown in Figure 7-11. The solid line shows the experiment and the dashed line shows the simulation. As can be seen that the experimental results of rotor1 and gimbal2 are similar to the simulation results, qualitatively in Figure 7, 8. The experimental result of gimbal4 is oscillated a little but it becomes stabilized in Figure 9. The control torques are shown in Figure 10, 11. The system can be controller within the limits. The proposed controller has usefulness for CMG control from the experiments.

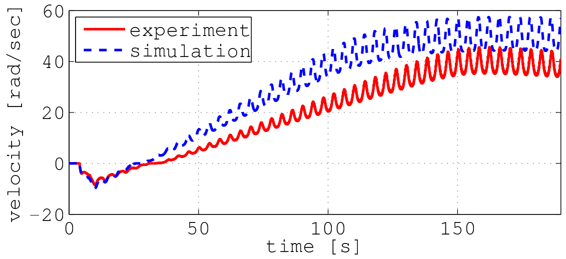

Figure 7 Experiment of ω_1

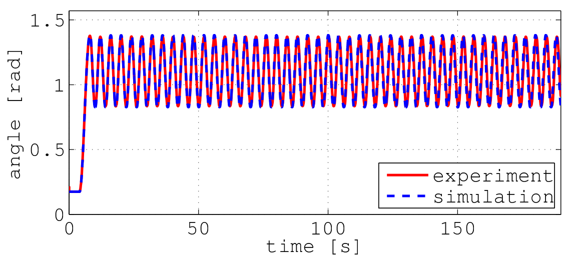

Figure 8 Experiment of q_2

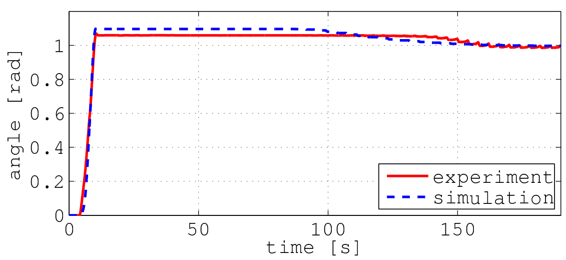

Figure 9 Experiment of q_4

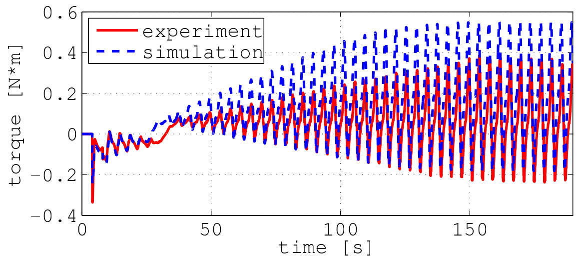

Figure 10 Experiment of torque T_1

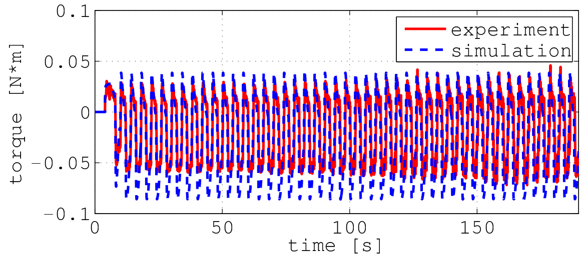

Figure 11 Experiment of torque T_2

6 Conclusion

In this research, a nonlinear tracking control of CMG to compensate the friction for nonholonomic system is proposed. Firstly chained form system from the equation of motion is derived. The state equation is converted into the chained system using ordinary algorithm. Secondly the tracking controller with integrator based on the backstepping method is designed. The integral of the error between the angle of gimbal4 and the reference trajectory is included to the controller. The integrator makes states track the reference without error despite the friction in the system. The stability of the system with integrator is guaranteed theoretically by consisting Lyapunov function. Finally the effectiveness of the proposed method is illustrated by simulations and experiments.

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