Linear Control of ABS without Using Approximation Around Equilibrium Point

M2015SC009 Yuki MARU
Supervisor: Isao TAKAMI

Abstract

In this paper, linear control of ABS (antilock braking system or antiskid braking system) without using approximation around equilibrium point is proposed. In conventional study of ABS using linear control, approximation around equilibrium point is used. It makes calculation easier. On the other hand, slip rate and friction coefficient will become constant. This may occur control error. Controller is designed to minimize the quadratic cost function, which is model of LQ controller. The robust stability for car velocity and friction coefficient between wheel and road surface are guaranteed by using polytopic representation. By using descriptor representation and linear fractional transformation(LFT), the system which is affine with respect to car velocity and friction coefficient can be obtained. Then, the problem is formulated as solving a finite set of Linear Matrix Inequalities (LMIs). Finally, the robust stability and the robust performance are guaranteed. The effectiveness of the proposed method is illustrated by simulations.

1 Introduction

An objective of ABS is to prevent car from occurring slip by wheel lock in brake operation at low friction road surface or when sudden baking. ABS was first developed for braking system of aircrafts(antiskid braking system). If aircraft make landing without ABS, wheels will wear out badly because of friction between road and the wheels. The wheels have to get changed often because it wears off quickly, and it cost a lot. If worse, the wheels will burst and an atrocious accident may happen. Because of these reasons, ABS was developed to give an optimal braking operation. This is that, until aircraft stops, the brake is operated not to stop the wheels perfectly. ABS(antilock braking system) loaded to cars are developed using aircraft's ABS. An objective of car's ABS is to prevent car from occurring slip by wheel lock in brake operation at low friction road surface or when sudden baking. It is well known that when the slip rate is nearly around 0.2, friction coefficient between wheel and road surface are high enough[1]. By keeping the slip rate to 0.2, braking distance and skidding can be prevented. Since the slip rate depends on car velocity and wheel velocity, ABS dynamic model depends on velocity and friction coefficient between wheel and the road surface.

Many study have been done about ABS such as nonlinear PID control[2], PID-type fuzzy control[3], and sliding mode control[4]. However, nonlinear control is not easy to be adapted. On the other hand, by adapting linear control and represent plant as state equation, it is able to design feedback system easily. Also, it is easy to evaluate designed controller. Study of ABS using linear control such as gain scheduling control[5] and LQ control[6] are done. In conventional study of ABS using linear control, approximation around equilibrium point is used. It makes calculation easier. On the other hand, slip rate and friction coefficient will become constant. This may occur control error.

In this paper, linear control of ABS without using approximation around equilibrium point is proposed. The robust stability for car velocity and friction coefficient between wheel and road surface are guaranteed by using polytopic representation. Then, the robust controller is obtained by solving a finite set of Linear Matrix Inequalities (LMIs). Finally, the effectiveness of the proposed method is illustrated by simulations.

2 Control Target and Modeling

2.1 Control Target

The model of the simplified ABS experimental device used in this study is shown in Fig.1.

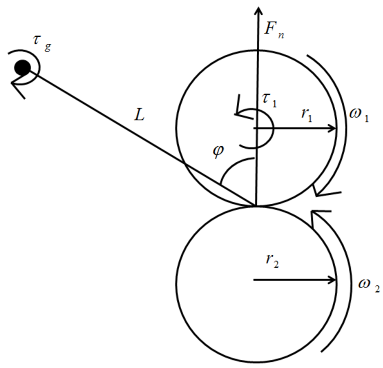

Figure 1 Simplified Diagram of the ABS Experimental Device

Table 1 The Physical Parameters

parameter	symbol	unit
Angular velocity of the upper wheel	ω_1	$[rad/s]$
Angular velocity of the lower wheel	ω_2	$[rad/s]$
Radius of the upper wheel	r_1	$[m]$
Radius of the lower wheel	r_2	$[m]$
Moment of inertia of upper wheel	J_1	$[kgm^2]$
Moment of inertia of lower wheel	J_2	$[kgm^2]$
Normal force	F_n	$[Nm]$
Brake torque	τ_1	$[Nm]$
Slip rate	λ	
Friction coefficient between wheels	$\mu(\lambda)$	

It is the quarter car model. The upper wheel simulates the car wheel, and the lower wheel simulates the road surface. Slip rate λ is taken as an output, and the brake torque τ_1 is taken as an input. By controlling the brake torque, control law is designed to keep slip rate at the optimal rate 0.2. To design the control law, differential equation expressing the dynamics of slip rate is required. In this section, the process to derive the linearized differential equation is shown.

2.2 Dynamical Equation

The dynamical equations of the rotational motion of the upper and lower wheels are shown by Eq(1) and Eq(2).

$$J_1\dot{\omega}_1 = F_n r_1 \mu(\lambda) - \tau_1 \quad (1)$$
$$J_2\dot{\omega}_2 = -F_n r_2 \mu(\lambda) \quad (2)$$

The slip rate is defined by Eq(3) as the function of car velocity and wheel velocity.

$$\lambda = \frac{r_2\omega_2 - r_1\omega_1}{r_2\omega_2} = \frac{V - V_w}{V} \quad (3)$$

The following equation is obtained from Eq(3).

$$\dot{\lambda} = -\frac{1}{V}\dot{V}_w + \frac{V_w}{V^2}\dot{V} \quad (4)$$

From Eq(1), Eq(2), Eq(3), and Eq(4), Eq(5) is derived.

$$\dot{\lambda} = -\frac{1}{V}(d_1 + d_2\lambda)\mu(\lambda) - \frac{1}{V}d_3\tau_1 \quad (5)$$
$$d_1 = \frac{F_n r_1}{J_1} + \frac{F_n r_2}{J_2}, d_2 = -\frac{F_n r_2}{J_2}, d_3 = -\frac{1}{J_1}$$

In this study, friction coefficient $\mu(\lambda)$ is given as polynomial approximation Eq(6)

$$\begin{aligned}\mu(\lambda) &= \alpha\frac{\{p_0 + p_1(\lambda - a) + p_2(\lambda - a)^2\}}{1 + q_1(\lambda - a) + q_2(\lambda - a)^2} \\ &= \alpha\frac{c_1 + c_2\lambda + p_2\lambda^2}{c_3 + c_4\lambda + q_2\lambda^2} \quad (6)\end{aligned}$$

The following equation is given by substituting Eq(6) to Eq(5).State variable $x(t)$ and input $u(t)$ are given as $x(t) = \lambda$,$u(t) = \tau_1$

$$\dot{\lambda} = Ax(t) + Bu(t) \quad (7)$$
$$A = -\alpha\frac{d_1c_2 + d_2c_1 + (d_1p_2 + d_2c_2)\lambda + d_2p_2\lambda^2}{V(c_3 + c_4\lambda + q_2\lambda^2)}$$
$$B = -\frac{1}{V}d_3$$

2.3 Feedforward

In this study, target-value tracking control using feedforward is considered. Let's consider the following system.

$$\begin{cases}\dot{x}(t) = Ax(t) + Bu(t) \\ y^{ref}(t) = Cx(t)\end{cases} \quad (8)$$

Controller Eq(9) which adding feedforward from target-value realize tracking control regarding target-value $y^{ref}(t) = {y_c}^{ref}$.

$$u(t) = Kx(t) + Hy^{ref}(t) \quad (9)$$

Let $y^{ref}(t) = {y_c}^{ref}$,$x(t) = x^*$,$u(t) = u^*$, and consider steady-state value condition $\dot{x}^* = 0$ in system(8)

$$\begin{cases}0 = Ax^* + Bu^* \\ {y_c}^{ref} = Cx^*\end{cases} \quad (10)$$

system(10) can be rewritten as follow.

$$\begin{bmatrix} A & B \\ C & 0 \end{bmatrix}\begin{bmatrix} x^* \\ u^* \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}{y_c}^{ref} \quad (11)$$

Here, ${y_c}^{ref} = x^*$, $C = 1$,$u^* = -\frac{A}{B}x^*.x^*$,$u^*$ which satisfy system(11)is given as follow.

$$\begin{aligned}\begin{bmatrix} x^* \\ u^* \end{bmatrix} &= \begin{bmatrix} A & B \\ C & 0 \end{bmatrix}^{-1}\begin{bmatrix} 0 \\ 1 \end{bmatrix}{y_c}^{ref} \\ &= \begin{bmatrix} 1 \\ u^* \end{bmatrix}{y_c}^{ref} \quad (12)\end{aligned}$$

Let define fluctuation from steady-state value x^*,u^* as $\tilde{x}(t) = x(t) - x^*$,$\tilde{u}(t) = u(t) - u^*$

$$\tilde{u}(t) = K\tilde{x}(t)$$
$$u(t) = K\tilde{x}(t) + u^* \quad (13)$$

From Eq.(13), let new state variable and input be $\tilde{x}(t) = \lambda - \lambda^*$,$\tilde{u}(t) = \tau_1 - \tau_1^*$.

2.4 State Space Representation

In order to track the output of the system to the optimal value without error, one integrator is added to the state variable. Let state variable be $\tilde{x}_e(t) = [\int(\lambda - \lambda^*)dt \quad \lambda - \lambda^*]^T$. Then, the state equation is obtained as follows.

$$E\dot{\tilde{x}}_e(t) = A_e\tilde{x}_e(t) + B_e\tilde{u}(t) \quad (14)$$
$$E = \begin{bmatrix} 1 & 0 \\ 0 & V(c_3 + c_4\lambda + q_2\lambda^2) \end{bmatrix}$$
$$A_e = \begin{bmatrix} 0 & 1 \\ 0 & -\alpha\{d_1c_2 + d_2c_1 + (d_1p_2 + d_2c_2)\lambda + d_2p_2\lambda^2\} \end{bmatrix}$$
$$B_e = \begin{bmatrix} 0 \\ -d_3(c_3 + c_4\lambda + q_2\lambda^2) \end{bmatrix}$$

2.5 Descriptor representation

To use polytopic representation, descriptor representation is applied to Eq(14). Let descriptor variable be $\tilde{x}_d(t) = [\tilde{x}_e(t)^T \quad \dot{\lambda} \quad u(t)]$ and derive the following descriptor equation.

$$\dot{\tilde{x}}_d(t) = A_d\tilde{x}_d(t) + B_d\tilde{u}(t) \quad (15)$$
$$E_d = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$
$$A_d = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & A_{d32} & A_{d33} & A_{d34} \\ 0 & 0 & 0 & -1 \end{bmatrix}$$
$$B_d = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}^T$$
$$A_{d32} = -\alpha\{d_1c_2 + d_2c_1 + (d_1p_2 + d_2c_2)\lambda + d_2p_2\lambda^2\}$$
$$A_{d33} = -V(c_3 + c_4\lambda + q_2\lambda^2)$$
$$A_{d34} = -d_3(c_3 + c_4\lambda + q_2\lambda^2)$$

2.6 Linear Fractional Transformation(LFT)

Only matrix A_d has uncertain parameters. However, high order terms of slip rate λ exist. So, LFT is applied to transform high order terms of slip rate λ to first order term. Matrix A_d can be represented by Eq(16). Here, A_n is the matrix which contains first order terms of λ, and $B_\delta(I-\Delta D_\delta)^{-1}\Delta C_\delta$ is the matrix which contains high order term of λ.

$$\begin{aligned} A_d &= A_n + B_\delta(I-\Delta D_\delta)^{-1}\Delta C_\delta \quad (16)\\ A_n &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & A_{n32} & A_{n33} & A_{n34} \\ 0 & 0 & 0 & -1 \end{bmatrix} \\ B_\delta &= \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}^T \\ C_\delta &= \begin{bmatrix} 0 & -\alpha d_2 p_2 \lambda & -V q_2 \lambda & -d_3 q_2 \lambda \end{bmatrix} \\ D_\delta &= 0, \Delta = \lambda \\ A_{n32} &= -\alpha\{d_1 c_2 + d_2 c_1 + (d_1 p_2 + d_2 c_2)\lambda\} \\ A_{n33} &= -V(c_3 + c_4\lambda) \\ A_{n34} &= -d_3(c_3 + c_4\lambda) \end{aligned}$$

Eq(15) can be expressed as Eq(17) by using A_n, B_δ, C_δ, D_δ, and Δ.

$$\begin{aligned} E_d\dot{\tilde{x}}_d &= A_n\tilde{x}_d + B_\delta w_\delta + B_d\tilde{u} \\ Z_\delta &= C_\delta\tilde{x}_d + D_\delta w_\delta \quad (17)\\ w_\delta &= \Delta z_\delta \end{aligned}$$

Let $\tilde{x}_l(t) = [\tilde{x}_d(t)^T \;\; Z_\delta(t)]^T$ be new descriptor variable, and the new descriptor equation be Eq.(18).

$$E_l\dot{\tilde{x}}_l(t) = A_l\tilde{x}_l(t) + B_l\tilde{u}(t) \quad (18)$$

$$E_l = \begin{bmatrix} E_d & 0 \\ 0 & 0 \end{bmatrix}, B_l = \begin{bmatrix} B_d \\ 0 \end{bmatrix}$$

$$A_l = \begin{bmatrix} A_n & B_\delta\Delta \\ C_\delta & D_\delta\Delta - I \end{bmatrix}$$

3 Controller Design

3.1 LQ controller

About linear state equation

$$\dot{x}(t) = Ax(t) + Bu(t), \quad (19)$$

state feedback controller be

$$u(t) = Kx(t), \quad (20)$$

close loop system is as follows.

$$\dot{x}(t) = (A + BK)x(t) \quad (21)$$

Following cost function for upper equation is considered

$$J = \int_0^\infty (x(t)^TQx(t) + u(t)^TRu(t))dt \quad (22)$$

Here, Q,R are weight matrices as follows.

$$Q = Q^T \succeq 0\ ,\ R = R^T \succ 0 \quad (23)$$

3.2 Polytopic Representation

Slip rate λ and car velocity V are nonlinear variables. In this study, robust stability is considered. Fluctuation range of slip rate λ and car velocity V are given as follow. Also, α is given as polytopic representation.

$$\Theta = \{[\theta_1,\ \theta_2,\ \theta_3]:\ \theta_i \in \{\underline{\theta_i},\ \overline{\theta_i}\}\}(i = 1,2,3)$$
$$\theta_1 = \lambda,\ \theta_2 = V,\ \theta_3 = \alpha$$
$$\Theta_1 = (\underline{\theta_1},\underline{\theta_2},\underline{\theta_3}),\ \Theta_2 = (\overline{\theta_1},\underline{\theta_2},\underline{\theta_3}),\cdots,\Theta_8 = (\overline{\theta_1},\overline{\theta_2},\overline{\theta_3})$$

3.3 LMI Condition

From 3.1 and 3.2, if there exist X_l and Y_l satisfying the following LMI conditions, the system is stabilized by gain $K_l = Y_l X_l{}^{-1}$ [6].

$$\begin{aligned} &minimize : \gamma \\ &subject\ to\ : X_l \succ 0 \quad (24)\\ &\begin{bmatrix} -He\{A_l(\Theta_i)X_l + B_lY_l\} & X_l(Q^{\frac{1}{2}})^T & Y_l^TR^T \\ Q^{\frac{1}{2}}X_l & I & 0 \\ RY_l & 0 & R \end{bmatrix} \\ &\succ 0 \ \ (i = 1,2,...,8) \quad (25)\\ &\begin{bmatrix} W & I \\ I & X \end{bmatrix} \succ 0 \quad (26)\\ &trace(W) < \gamma \quad (27)\\ &X_l = \begin{bmatrix} X & 0 & 0 \\ X_{21} & X_{22} & X_{23} \\ X_{31} & X_{32} & X_{33} \end{bmatrix}, Y_l = \begin{bmatrix} Y & 0 & 0 \end{bmatrix} \end{aligned}$$

4 Simulation

In this section, comparison between proposed model and conventional model is illustrated by simulation. In the simulation, robust LQ controller is used. Here, conventional model is given first, then weight matrices for both models, and finally simulations are illustrated.
State space equation of conventional model is given as below. Here, $T = \frac{1}{V}$.

$$\dot{\tilde{x}}_e(t) = A_c\tilde{x}_e(t)B_c\tilde{u}(t) \quad (28)$$

$$A_c = \begin{bmatrix} 0 & 1 \\ 0 & -T\{(d_1 + d_2\lambda^*)\frac{d}{d\lambda}\mu(\lambda^*) + d_2\mu(\lambda^*)\} \end{bmatrix}$$

$$B_c = \begin{bmatrix} 0 \\ -Td_3 \end{bmatrix}$$

Parameter box is given as follows.

$$\Theta = \{[\theta_1,\ \theta_2]:\ \theta_i \in \{\underline{\theta_i},\ \overline{\theta_i}\}\}(i = 1,2) \quad (29)$$
$$\theta_1 = T,\ \theta_2 = \alpha$$
$$\Theta_1 = (\underline{\theta_1},\underline{\theta_2}),\ \Theta_2 = (\overline{\theta_1},\underline{\theta_2}),\ \Theta_3 = (\underline{\theta_1},\overline{\theta_2}),\ \Theta_4 = (\overline{\theta_1},\overline{\theta_2})$$

LMI conditions are given as follows.If there exist X_l and Y_l satisfying the LMI conditions below, the system is stabilized by gain $K = Y_cX_c^{-1}$ [6].

$$\begin{aligned} &minimize : \gamma \\ &subject\ to\ : X \succ 0 \quad (30)\\ &\begin{bmatrix} -He\{A_c(\Theta_i)X_c + B_cY_C\} & X_c(Q_c^{\frac{1}{2}})^T & Y_c^TR^T \\ Q_c^{\frac{1}{2}}X_c & I & 0 \\ RY_c & 0 & R \end{bmatrix} \succ 0 \\ &(i = 1,2,3,4) \quad (31)\\ &\begin{bmatrix} Z & I \\ I & X \end{bmatrix} \succ 0 \quad (32)\\ &trace(W) < \gamma_c \quad (33) \end{aligned}$$

Weight matrices and the upper bound of the cost function of proposed model are as follows.

$$Q = diag([\ 100 \quad 10 \quad 0 \quad 0 \quad 0\ ]),\ R = 0.01$$
$$\gamma = 33.15654$$

Weight matrices and the upper bound of the cost function of conventional model are as follows.

$$Q_c = diag([\ 100 \quad 10\ ]),\ R = 0.01$$
$$\gamma_c = 31.92369$$

In this simulation, braking starts from 50[km/h] on dry road and frozen road.

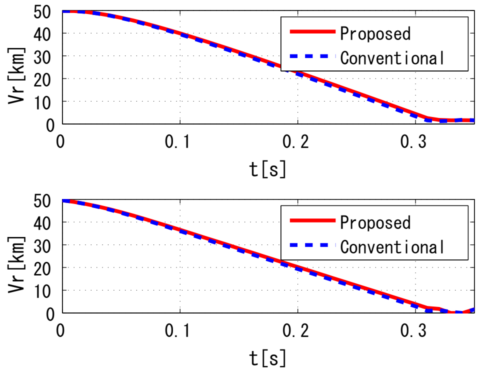

Figure 2 Car Velocity On Dry Road

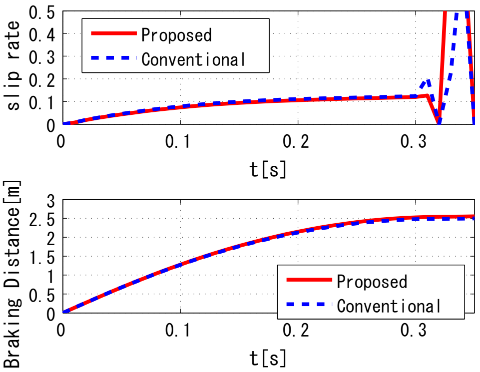

Figure 3 Car Velocity On Dry Road

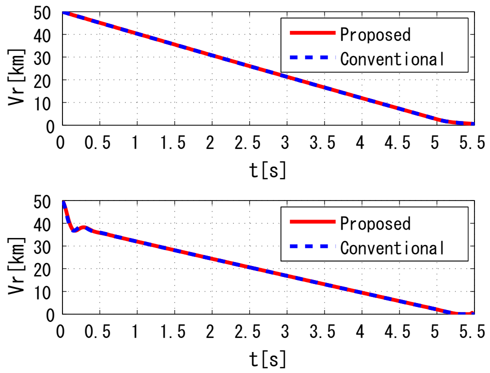

Figure 4 Car Velocity On Dry Road

5 Conclusion

In this study, linear control of ABS without using approximation around equilibrium point is proposed.

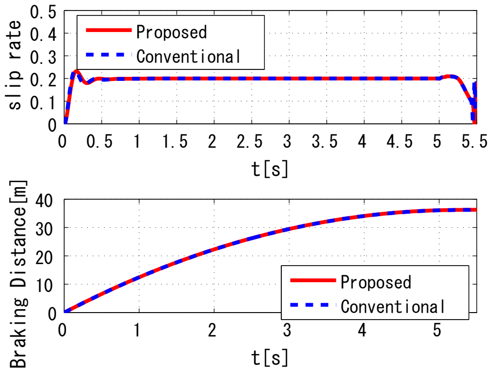

Figure 5 Car Velocity On Dry Road

Since the car velocity and friction coefficient affects the dynamics, a method is designed to consider variation of these parameters. To linearize the model, descriptor representation and linear fractional transformation (LFT), polytopic representation is applied for ABS. Friction coefficient is given as function of slip rate λ by polynomial approximation. The process to obtain dynamical model, controller, and LMI conditions are shown in this paper. From simulation, slip rate is controlled around the optimal value 0.2 on frozen road. By comparing conventional model and proposed model, the result is almost the same. However, on dry road, braking distance of conventional model was a little shorter than proposed model. It is presumed that by using descriptor representation and LFT to proposed model, conservativeness get higher and readiness decreased.

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