

Design of Circularly Polarized Dielectric Resonator Antennas using Modal Polarization Current Model Method

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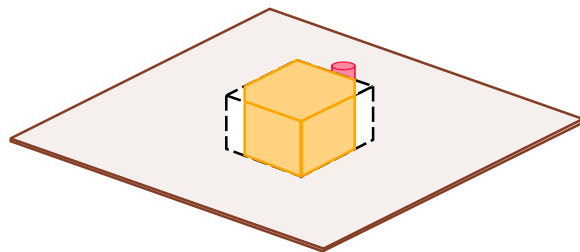


Introduction

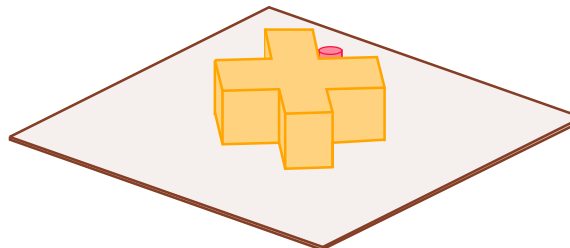
Circularly Polarized Dielectric Resonator Antenna (CP-DRA) With Single Probe Feed

Merit : *Simple Feed Structure*

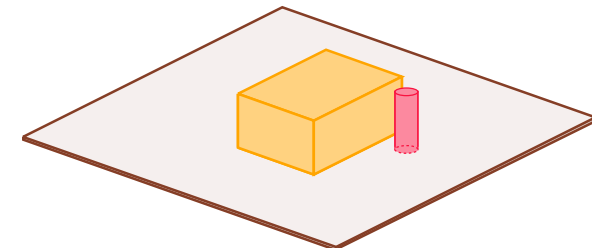
Demerit : *Narrow Frequency Bandwidth*



Notch Added Shape



Cross Shape



Cuboidal Shape



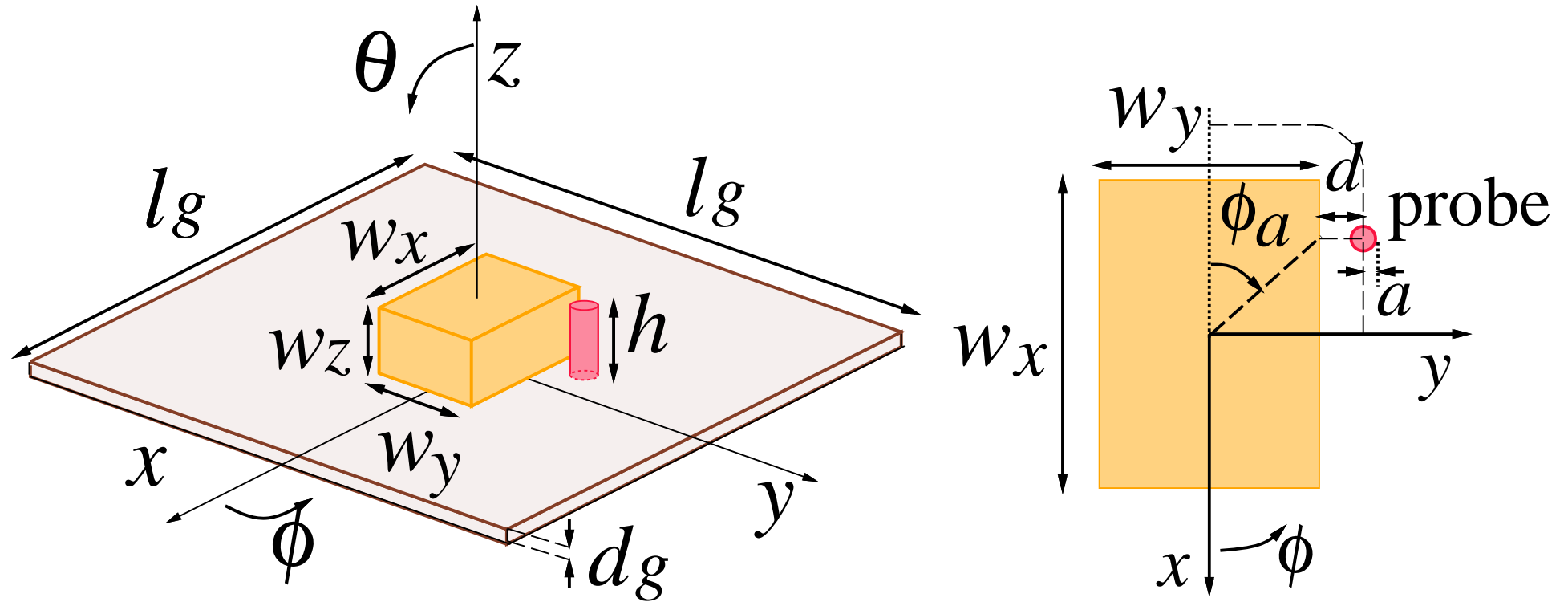
Cuboid CP-CDR

- *Easy in Fabricating Dielectrics*
- *CP Excitatin Condition Not Established*

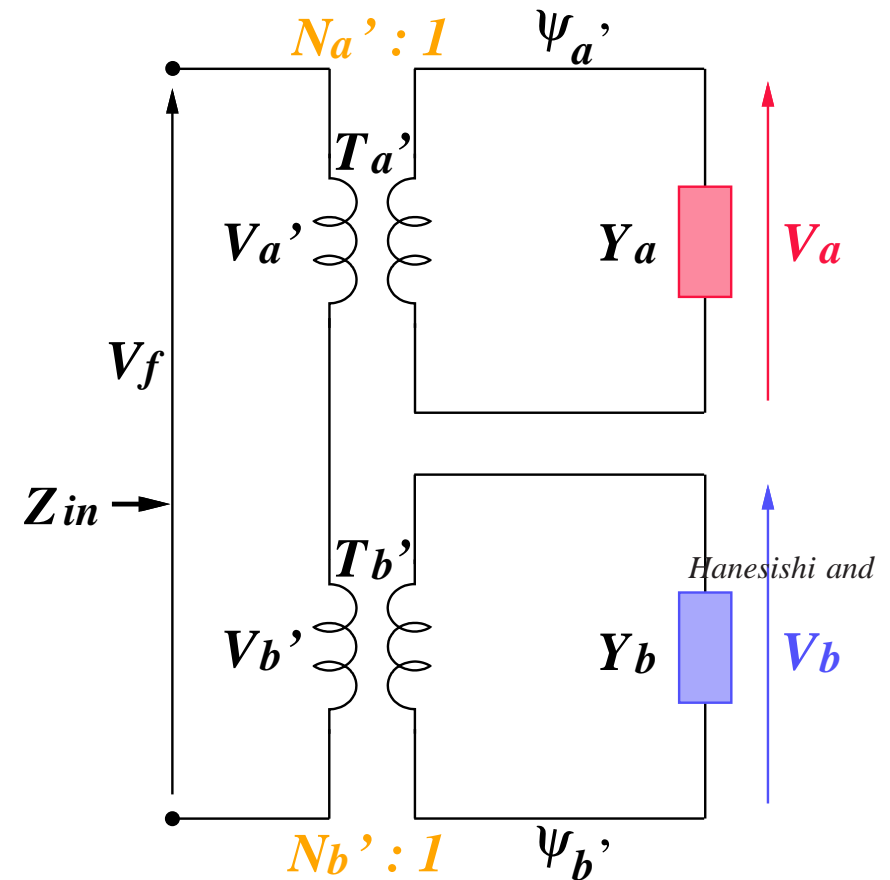
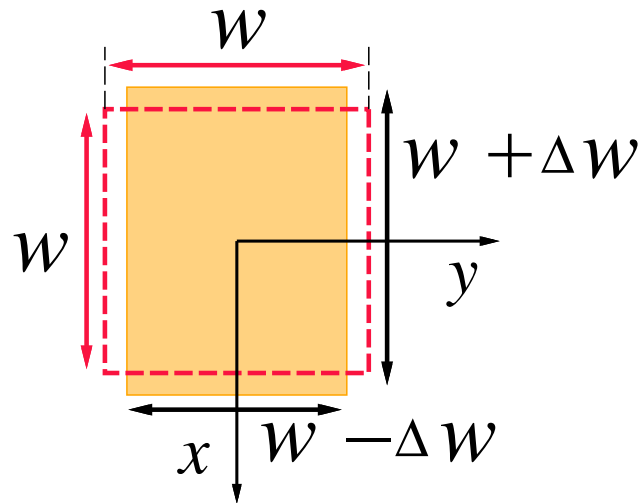
Objective

Design of CP-PDRA Composed of Single Feed and Cuboidal Dielectrics

1. Application of ECM(Equivalent Circuit Method) for CP-Microstrip Antenna to CP-DRA
2. Analyze the CP Characteristics Dependence on Perturbation, Feed Position, Probe Length, by M-PCMM(Modal Polarization Current Model Mehtod)
3. Establish the Design Method of CP-DRA with Circular Polarization and Impedance Matching
4. Prove by Experimentt the Proposed Design Method

STRUCTURE AND COORDINATE

EQUIVALENT CIRCUIT WHEN PERTURBED



$$Y_a = G_0 \left\{ 1 + j Q_0 \left(\frac{f}{f_a} - \frac{f_a}{f} \right) \right\}$$

$$Y_b = G_0 \left\{ 1 + j Q_0 \left(\frac{f}{f_b} - \frac{f_b}{f} \right) \right\}$$

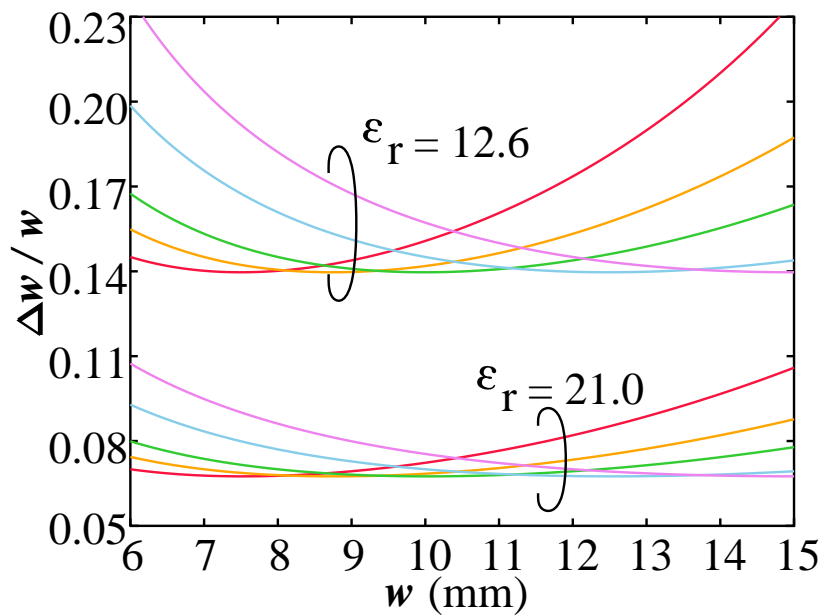
$$N_a' = \sqrt{2V} \left\{ 1 - \frac{1}{2} (p_{11} - p_{12}) \right\} A \sin k_w x \cos k_s y \cos k_z z$$

$$N_b' = \sqrt{2V} \left\{ 1 - \frac{1}{2} (p_{11} + p_{12}) \right\} A \cos k_s x \sin k_w y \cos k_z z$$

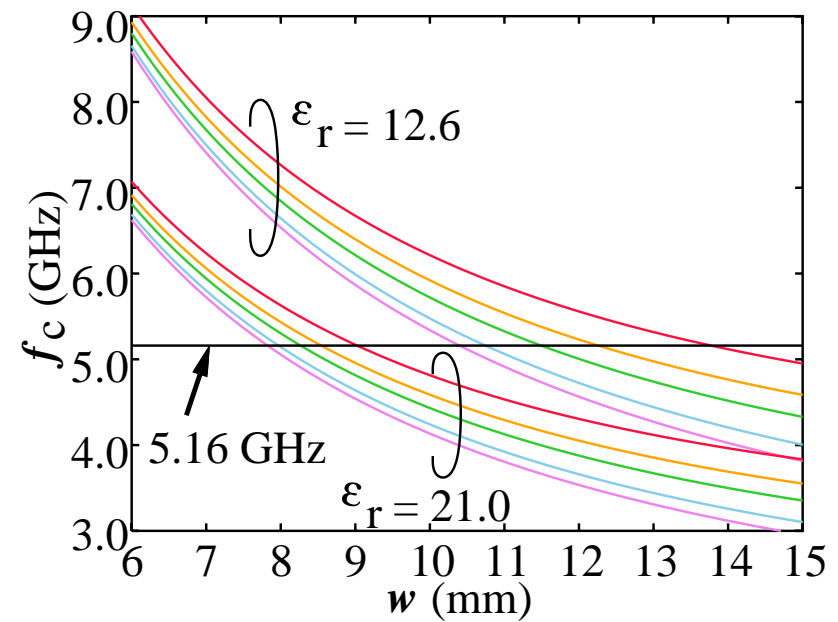
Yoshida , " A Method of Designing Singly Fed Rectangular CP Microstrip Antenna" EICE Trans.(B) , vol.J64-B , no.4 , pp.225-231 , Apr. 1981.

DESIGN PARAMETERS OBTAINED BY ECM

$w_z = 6$ mm — red — $w_z = 10$ mm — light blue —
 $w_z = 7$ mm — orange — $w_z = 12$ mm — magenta —
 $w_z = 8$ mm — green —



w vs. $\Delta w/w$



w vs. f_c

APPLICATION OF M-PCMM

Equivalent Circuit Method(ECM)

Merit Possible to obtain the dielectrics parameters approximately minimizing the axial ratio at the usable frequency

Demerit Impossible to obtain the feed position or probe length for impedance matching



Necessary to include the feeding position and probe length in the design



Application of M-PCMM

Clarify the characteristics dependence on perturbation , feeding position , and probe length as CP-DRA

PCMM : Polarization Current Model Method

- **Method of Moment** using the polarization current (J. H. Richmond, 1964)
- Electromagnetic Field Analysis of the Space including Dielectrics
- Analysis of a Linear Antenna near a Dielectrics

M-PCMM : Modal PCMM

- Expand the polarization current inside the dielectrics by common spherical wave functions (entire domain MoM)
- Analysis of the space including a homogeneous dielectrics
- Applicable to sphere-like dielectrics difficult to analyze by volumetric integral equation method



Example of the application

: Dielectric Resonator Antennaf (DRA)

Inoue, Inagaki and Kikuma, " Application of Modal Polarization Current Model Method to Dielectric Resonator Antenna," IEICE Trans.(B), Vol.J86-B, no.1, pp.76-84, Jan. 2003.

Application of M-PCMM to DRA

	M-PCMM	HFSS	DWM
Resonant Frequency			
Input Impedance, Gain			×
Unloaded Q			
Directive Pattern			
Source Separation		×	×
Internal Field, Probe Current			×
Inhomogeneous Dielectrics	×		×
DRA Analysis with the parameters fixed			×
DRA Analysis with probe parameters varied			×



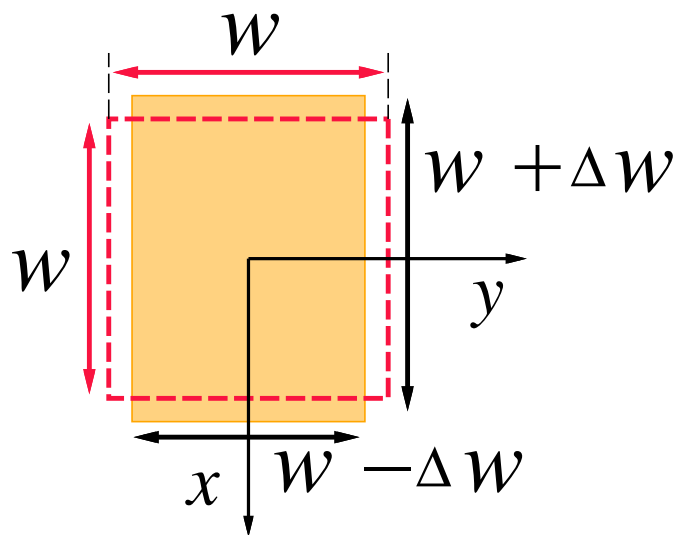
Useful in designing DRA

- Sufficient Accuracy consistent with measurement
- Efficient Analysis for variable feeding probe parameters

Perturbation $\Delta w/w$ and the dielectrics parameters

$$w_x = w + \Delta w$$

$$w_y = w - \Delta w$$



Frequency

5.15 ~ 5.25GHz

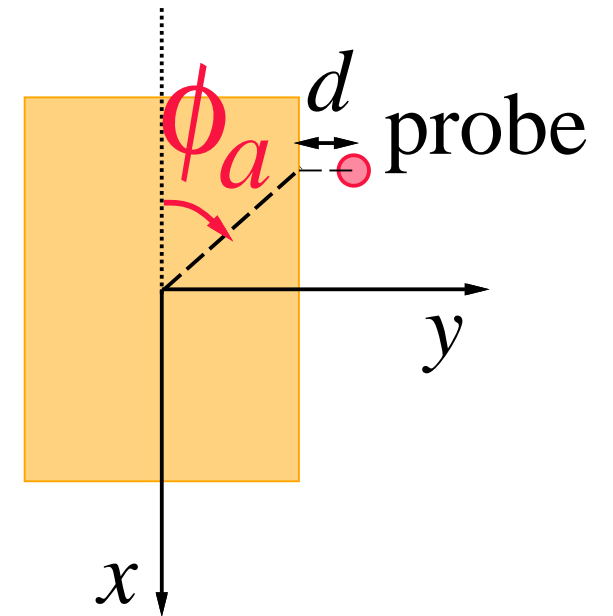
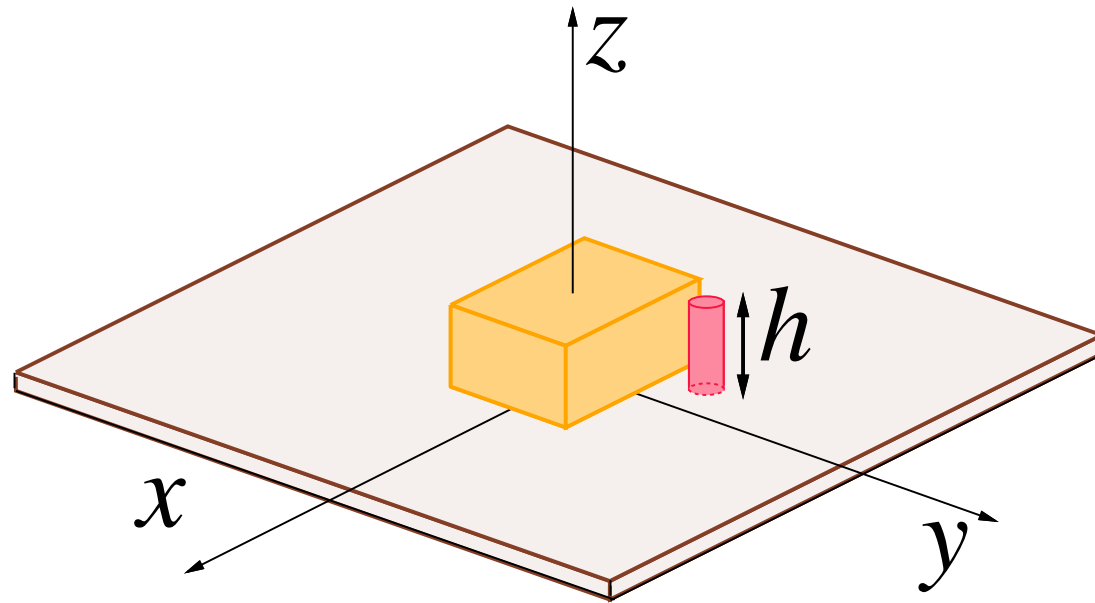
Radio LAN

IEEE802.11a standard

$$\epsilon_r = 12.6, w_z = 8.0\text{mm}, w = 11.5\text{mm}$$

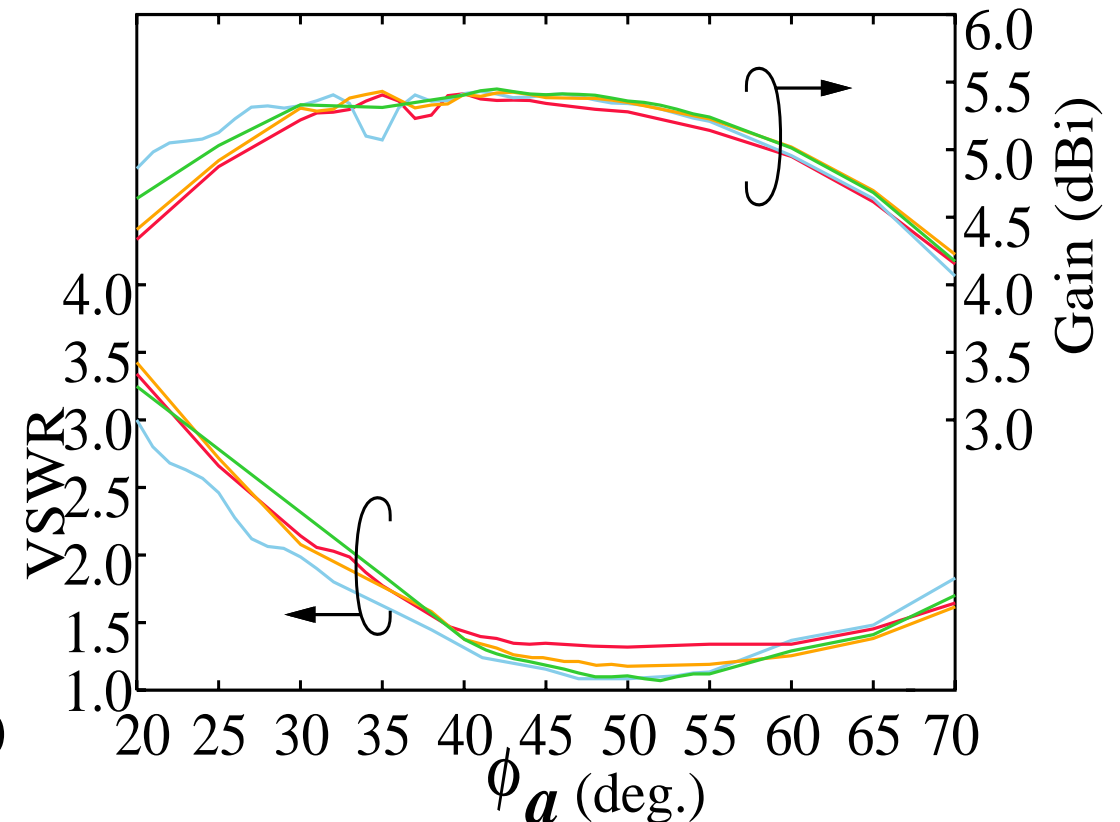
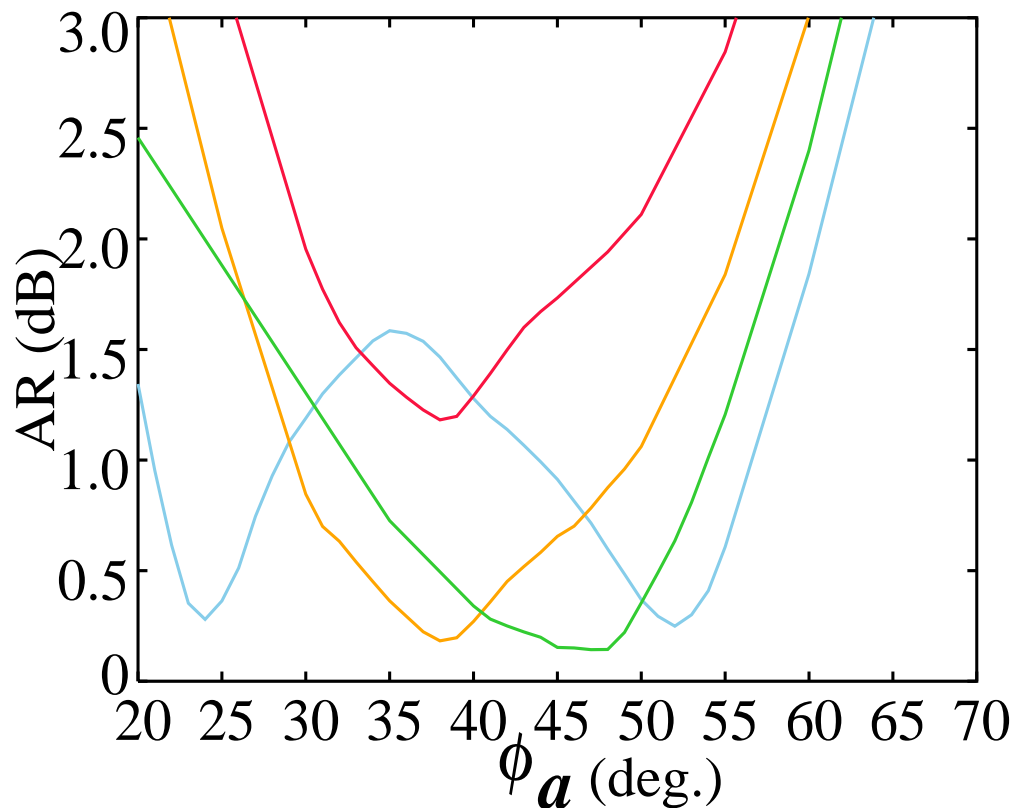
f_c (GHz)	$\Delta w/w$	w_x (mm)	w_y (mm)	w_z (mm)
5.159	0.130	13.0	10.0	8.0
5.160	0.142	13.13	9.87	8.0
5.162	0.160	13.34	9.66	8.0
5.164	0.180	13.57	9.43	8.0

Singly Fed Cuboidal CP-DRA



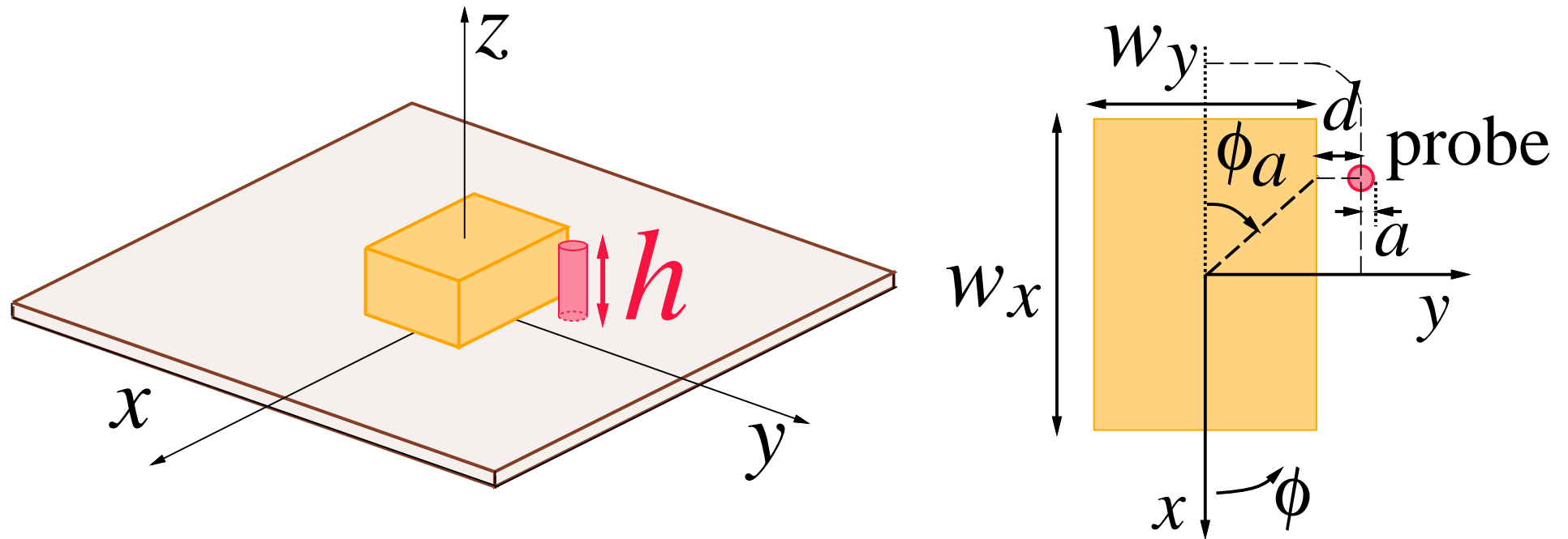
ϕ_a vs. CP-DRA characteristics

$\Delta w / w = 0.130$ — (red) $\Delta w / w = 0.160$ — (green)
 $\Delta w / w = 0.142$ — (orange) $\Delta w / w = 0.180$ — (blue)



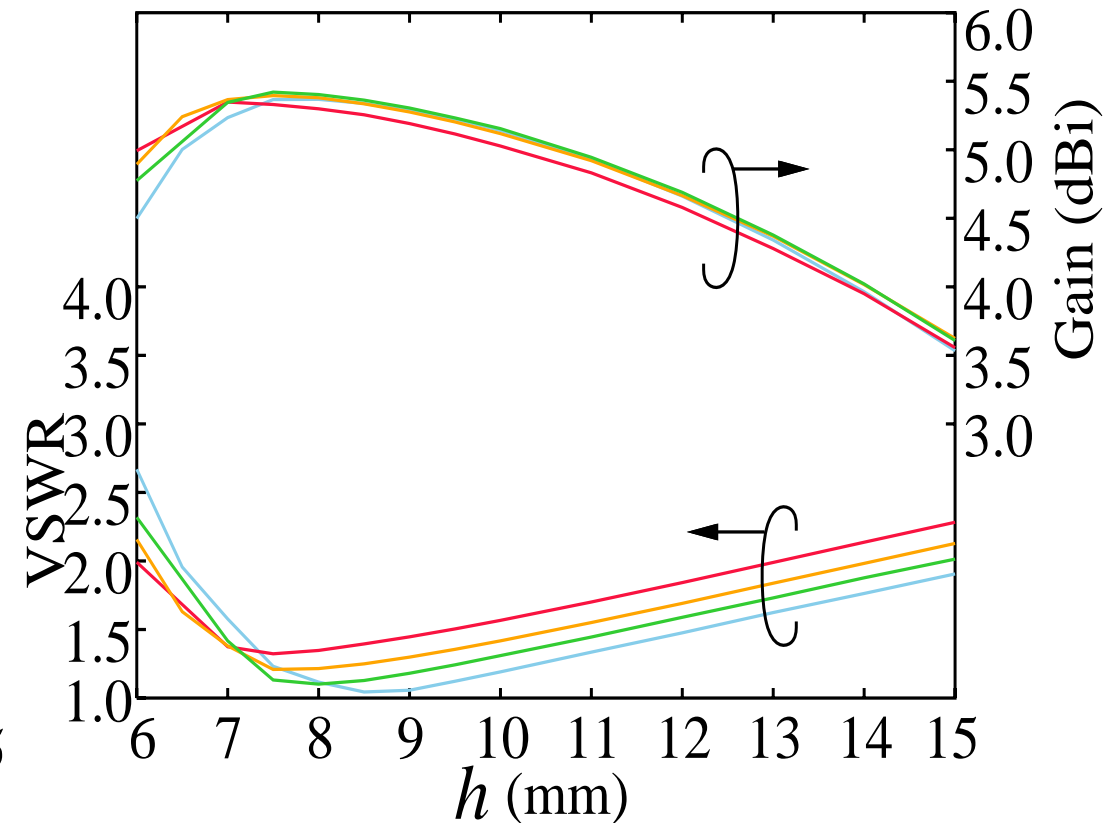
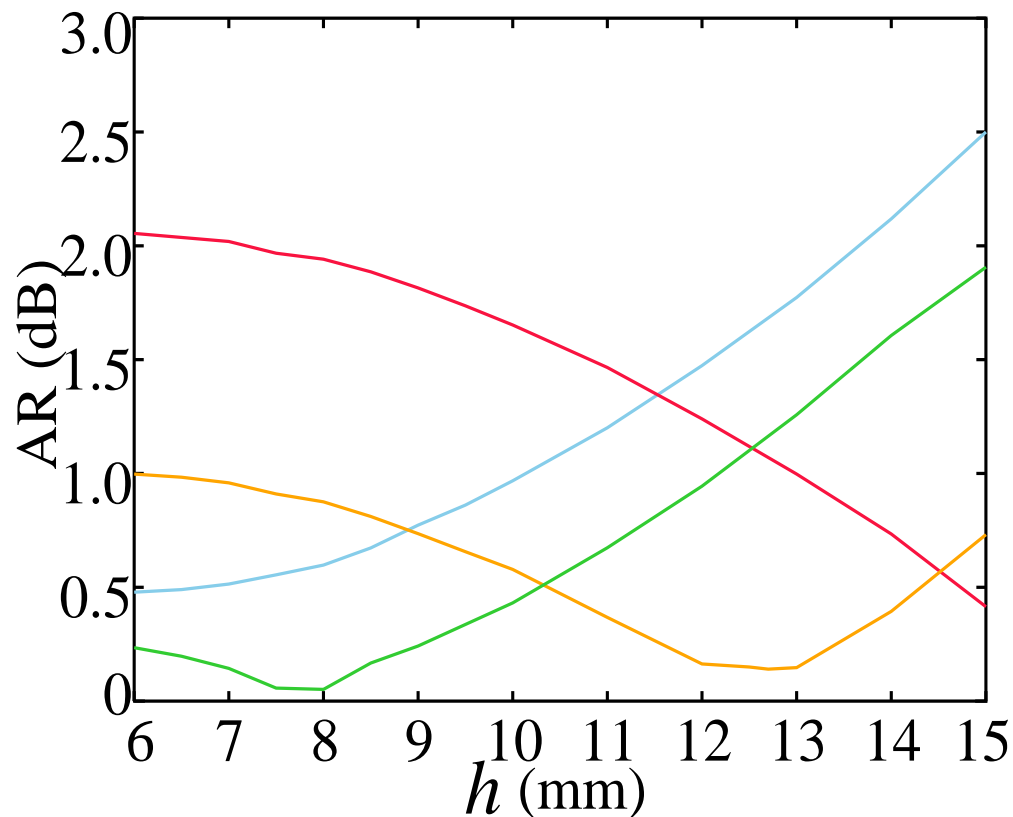
$\epsilon_r = 12.6$, $\tan \delta = 2.22 \times 10^{-5}$, $w_z = 8.0\text{mm}$, $w = 11.5\text{mm}$,
 $h = 8.0\text{mm}$, $d = 0.6\text{mm}$, $a = 0.5\text{mm}$

Cuboidal CP-DRA With Single Probe Feed



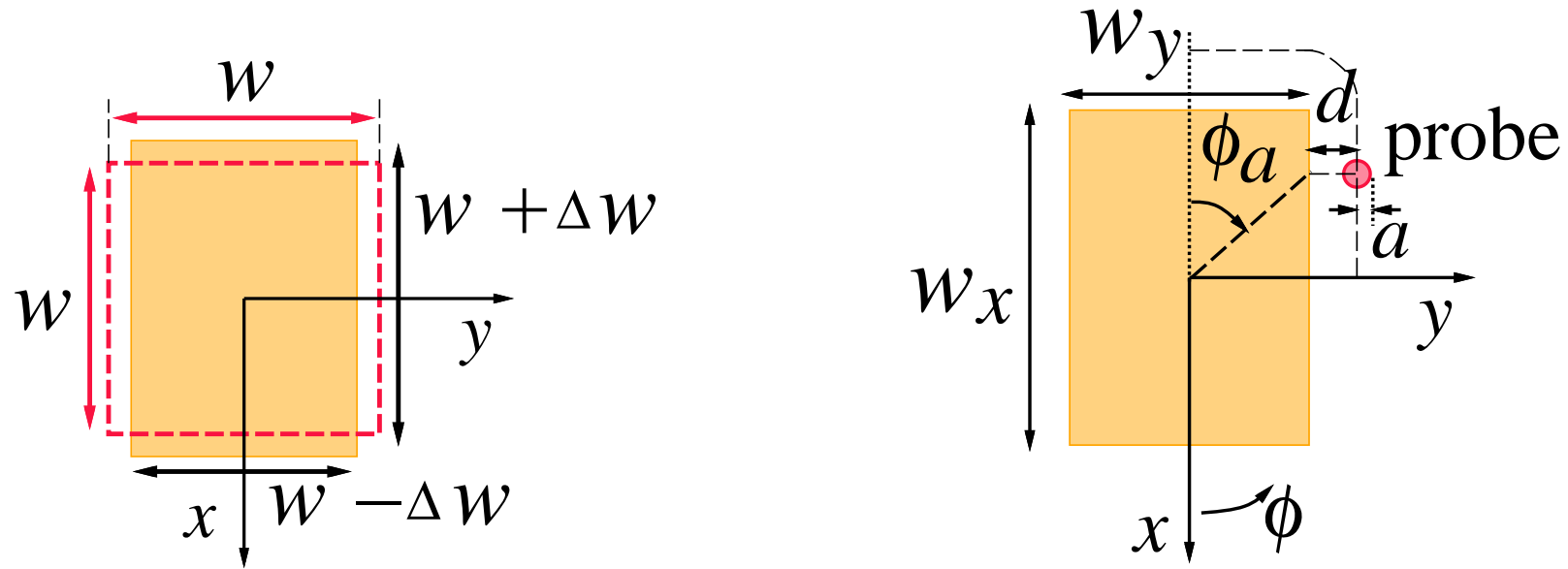
h vs. CP-DRA Characteristics

$\Delta w / w = 0.130$ — (red) $\Delta w / w = 0.160$ — (green)
 $\Delta w / w = 0.142$ — (orange) $\Delta w / w = 0.180$ — (blue)



$\epsilon_r = 12.6$, $\tan \delta = 2.22 \times 10^{-5}$, $w_z = 8.0\text{mm}$, $w = 11.5\text{mm}$,
 $\phi_a = 48^\circ$, $d = 0.6\text{mm}$, $a = 0.5\text{mm}$

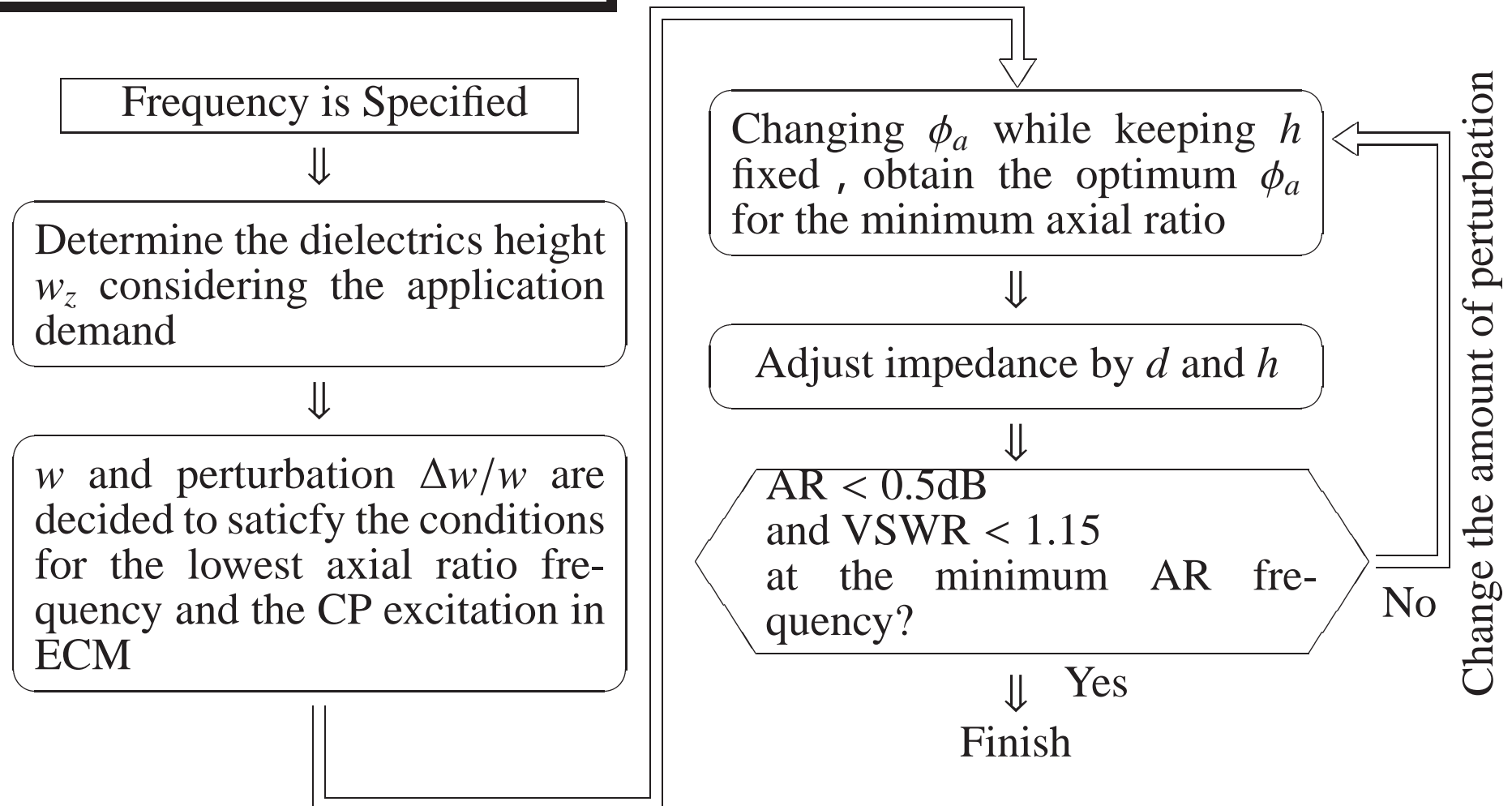
Perturbation, Probe Length and Position vs. CP-DRA Characteristics



	$\Delta w/w$	ϕ_a	d	h
AR				
Input Impedance, Gain				
f_c	×		×	×

Dependency : > > ×

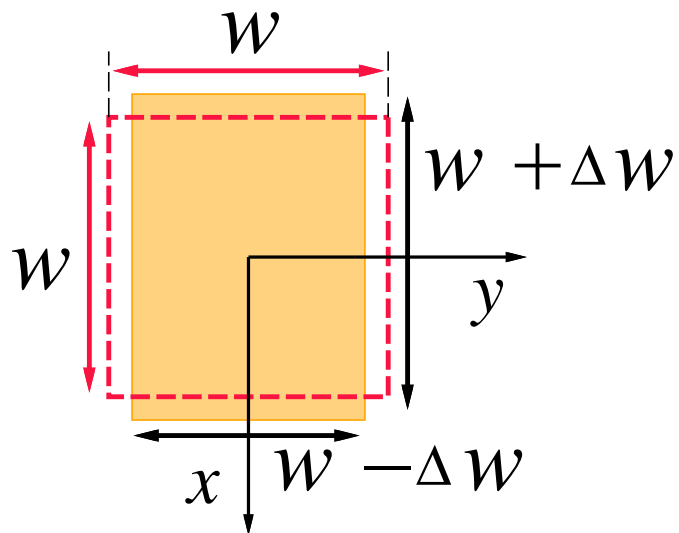
CP-DRA Design Procedure



Parameters Obtained at the Second Step of Design

$$w_x = w + \Delta w$$

$$w_y = w - \Delta w$$



frequency

5.15 ~ 5.25GHz

Radio LAN

IEEE802.11a standard

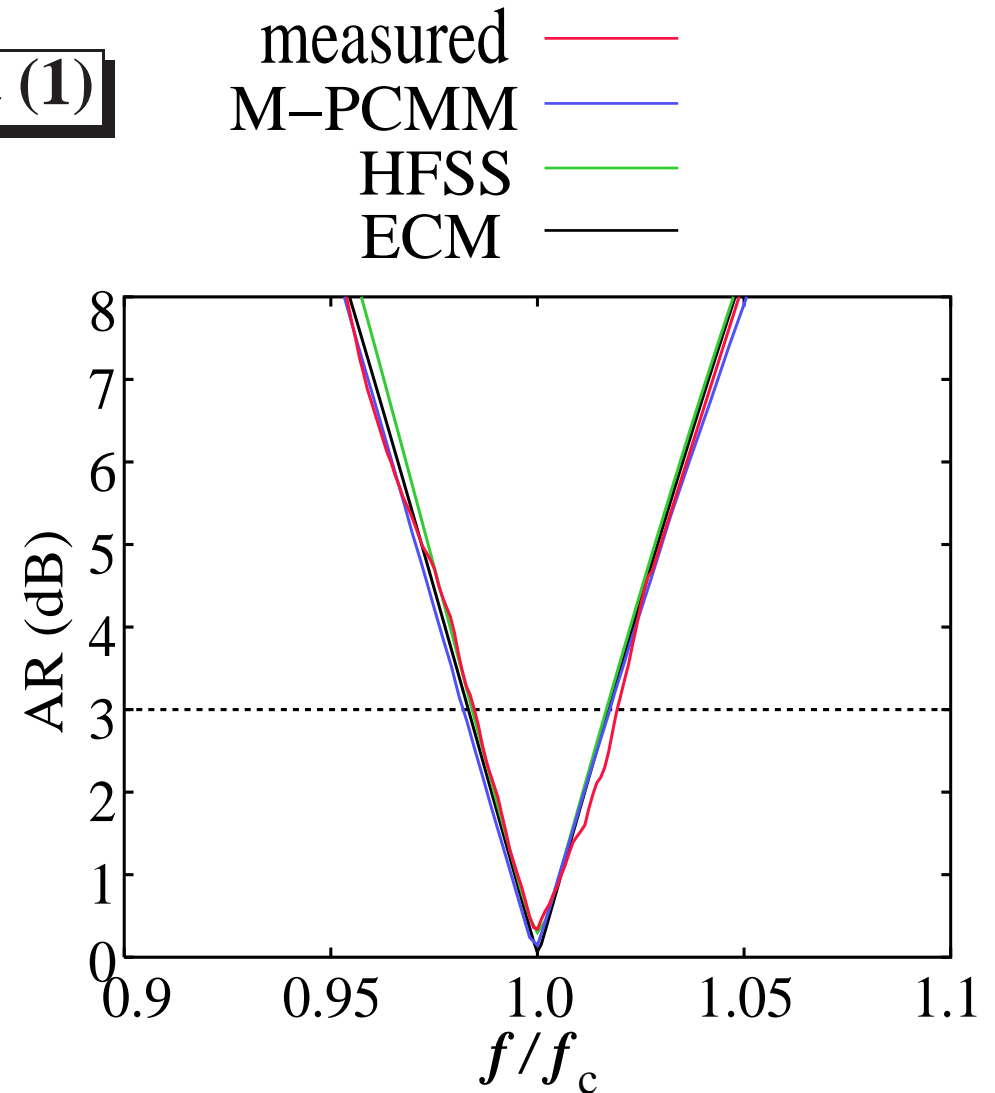
$$\epsilon_r = 12.6, w_z = 8.0\text{mm}, w = 11.5\text{mm}$$

f_c (GHz)	$\Delta w/w$	w_x (mm)	w_y (mm)	w_z (mm)
5.159	0.130	13.0	10.0	8.0
5.160	0.142	13.13	9.87	8.0
5.162	0.160	13.34	9.66	8.0
5.164	0.180	13.57	9.43	8.0

$$\phi_a = 48^\circ, h = 8.0\text{mm}, d = 0.6\text{mm}, a = 0.5\text{mm}$$

Comparison with the Measurement (1)

	f_c (GHz)	AR (dB)	AR width(%)
Measured	5.23	0.340	3.45
M-PCMM	5.28	0.144	3.57
HFSS	5.18	0.299	3.25
ECM	5.16	0	3.39



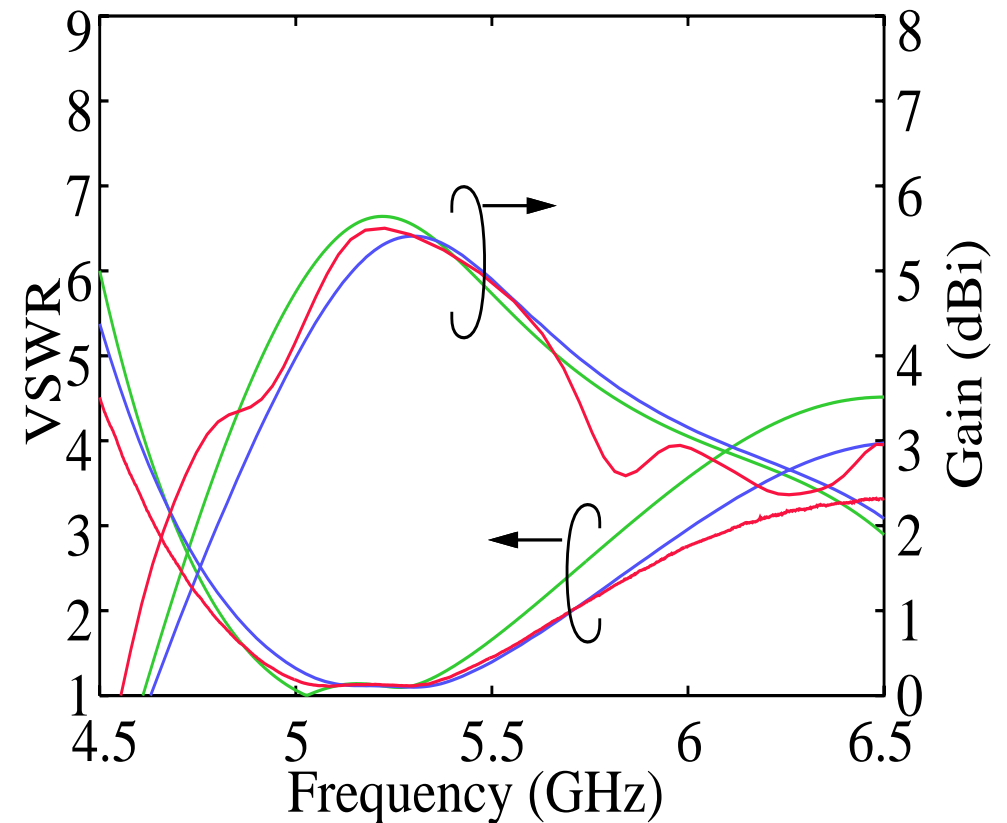
$$\varepsilon_r = 12.6, \tan \delta = 2.22 \times 10^{-5}, w_z = 8.0\text{mm}, w = 11.5\text{mm},$$

$$\Delta w/w = 0.16, \phi_a = 48^\circ, h = 8.0\text{mm}, d = 0.6\text{mm}, a = 0.5\text{mm}$$

Comparison with the Measurement (2)

measured — (red)
 M-PCMM — (blue)
 HFSS — (green)

	f_c (GHz)	VSWR	VSWR width(%)
Measured	5.23	1.13	17.6
M-PCMM	5.28	1.10	16.5
HFSS	5.18	1.14	15.2
ECM	5.16	—	—



$$\epsilon_r = 12.6, \tan \delta = 2.22 \times 10^{-5}, w_z = 8.0\text{mm}, w = 11.5\text{mm},$$

$$\Delta w/w = 0.16, \phi_a = 48^\circ, h = 8.0\text{mm}, d = 0.6\text{mm}, a = 0.5\text{mm}$$

Conclusion

Investigation of the Desing Method for CP-DRA

- Applied the ECM, the method used in CP Microstrip Antenna, to the CP-DRA
- Analyzed by M-PCMM the CP-DRA Characteristics Dependence on the Perturbation, Feeding Position, Probe Length

Desing of CP-DRA with Impedance Matching

At the first step, obtain the dielectrics parameters by ECM as initial values for the minimum axial ratio at the operation frequency, ant at the second step, by the **repeated analysis of the structures including the excitation** obtain the optimum structural parameters

- *Sufficient accuracy consistent with the measurement*
- *Efficient analysis for the parameter variations of the excitation structure*



M-PCMM is useful in the second step, where the feed is included

Thank you very much for your kind attention

M-PCMM (Modal Polarization Current Model Method)

Vector potential(r component) in terms of spherical wave functions

$$A_r = \sum_{n=1}^N \sum_{m=0}^n \hat{J}_n(k_d r) P_n^m(\cos \theta) \{ M_1^{nm} \cos m\phi + M_2^{nm} \sin m\phi \}$$

$$F_r = \sum_{n=1}^N \sum_{m=0}^n \hat{J}_n(k_d r) P_n^m(\cos \theta) \{ M_3^{nm} \cos m\phi + M_4^{nm} \sin m\phi \}$$

M_1^{nm} , M_2^{nm} : Current dimension

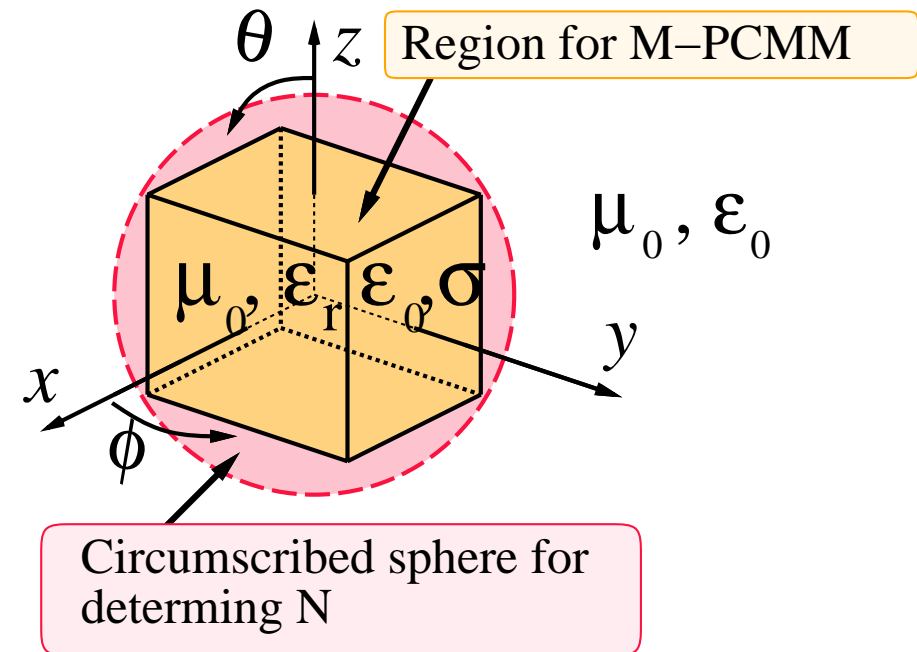
M_3^{nm} , M_4^{nm} : Voltage dimension

$P_n^m(x)$: Associated Legendre function of the first kind

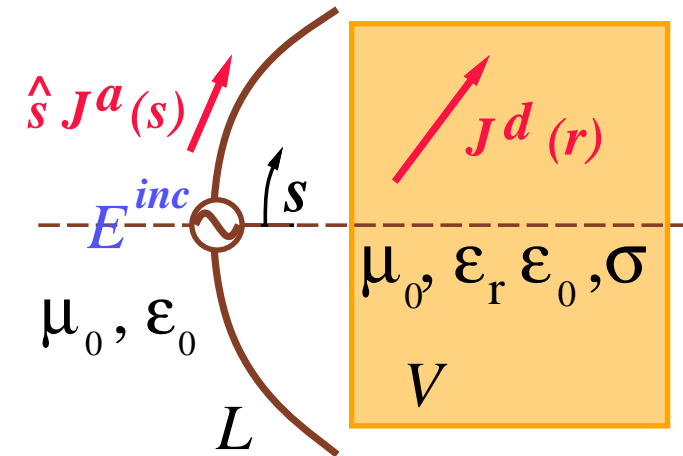
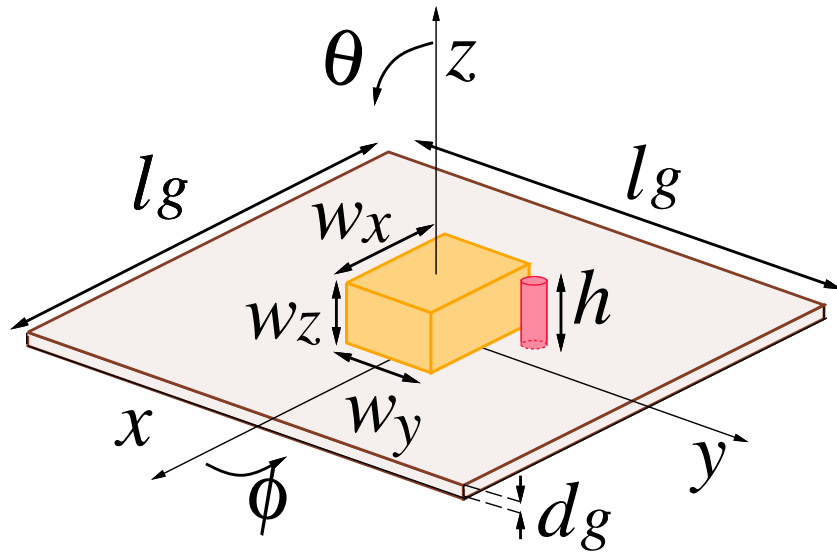
k_d : Wave number inside the dielectrics

$j_n(z)$: Spherical Bessel function of order n

$$\hat{J}_n(z) = z j_n(z)$$



Integral equation for the probe fed DRA



$$\hat{s} \cdot \int_{ant} J^a(s') \hat{s}' \cdot \bar{G}(s, s') ds' + \hat{s} \cdot \iiint_V \tau(\mathbf{r}') \mathbf{E}^d(\mathbf{r}') \cdot \bar{G}(s, \mathbf{r}') dV' = \delta(s) \frac{E^{inc}}{\Delta_{gap}}$$

$$- \int_{ant} J^a(s') \hat{s}' \cdot \bar{G}(\mathbf{r}, s') ds' + \left\{ 1 - j \frac{\eta_0}{3k_0} \tau(\mathbf{r}) \right\} \mathbf{E}^d(\mathbf{r}) - \text{PV} \iiint_V \tau(\mathbf{r}') \mathbf{E}^d(\mathbf{r}') \cdot \bar{G}(\mathbf{r}, \mathbf{r}') dV' = 0$$

$$\bar{G}(\mathbf{r}, \mathbf{r}') = -jk_0 \eta_0 \left(\bar{\mathbf{I}} + \frac{\nabla \nabla}{k_0^2} \right) \frac{\exp(-jk_0 |\mathbf{r} - \mathbf{r}'|)}{4\pi |\mathbf{r} - \mathbf{r}'|}$$

$\hat{s} J^a(s)$: Current on the probe

$\mathbf{J}^d(\mathbf{r}) = \tau(\mathbf{r}) \mathbf{E}^d(\mathbf{r})$: Polarization current density

$\mathbf{E}^d(\mathbf{r})$: Total electric field inside the dielectrics

$$\tau(\mathbf{r}) = [\sigma(\mathbf{r}) + j\omega\{\epsilon(\mathbf{r}) - \epsilon_0\}]$$

PV : Principal value of integral

Matrix equation by M-PCMM

$$\begin{array}{c} N_a \updownarrow \\ 3N_d \updownarrow \end{array} \begin{array}{c} \overleftrightarrow{N_a} \\ \overleftrightarrow{N_M} \end{array} \begin{pmatrix} \mathbf{Z}^{aa} & \mathbf{Z}^{aM} \\ \mathbf{Z}^{da} & \mathbf{Z}^{dM} \end{pmatrix} \begin{pmatrix} \mathbf{J}^a \\ \mathbf{M} \end{pmatrix} = \begin{pmatrix} \mathbf{V} \\ \mathbf{0} \end{pmatrix}$$

$$\{ (\mathbf{Z}^{aa}) - (\mathbf{Z}^{aM})(\mathbf{Z}^{MM})^{-1}(\mathbf{Z}^{Ma}) \} (\mathbf{J}^a) = (\mathbf{V})$$

$$(\mathbf{M}) = -(\mathbf{Z}^{MM})^{-1}(\mathbf{Z}^{Ma})(\mathbf{J}^a)$$

$$(\mathbf{Z}^{MM}) = (\mathbf{Z}^{dM})^\dagger (\mathbf{Z}^{dM})$$

$$(\mathbf{Z}^{Ma}) = (\mathbf{Z}^{dM})^\dagger (\mathbf{Z}^{da})$$

- (\mathbf{J}^a) : Probe current expansion coefficient column vector
- (\mathbf{M}) : mode expansion coefficient column vector
- (\mathbf{V}) : Incident voltage column vector

- Approximate the volumetric integral in the dielectrics by the sum of N_d volumetric integrals in the cuboid V_q
- Polarization current density is approximated to be uniform and equal to that at the center inside the devided region
- Apply the point-matching at the center of the devided regions
- For the observation point coincident with the source point, approximate the integral by the analytic one for the spherical region with the same volume(radius u_q) as the cuboid

$$\begin{aligned}
 & j\frac{\eta_0}{3k_0}\tau(\mathbf{r})\delta_{ij} + \text{PV} \iiint_{V_q} \tau(\mathbf{r}_q) \left(k_0^2 \delta_{ij} + \frac{\partial^2}{\partial x_i \partial x_j} \right) \frac{\exp(-jk_0|\mathbf{r}_p - \mathbf{r}'|)}{4\pi|\mathbf{r}_p - \mathbf{r}'|} dv' \\
 & = -j\frac{\eta_0}{k_0}\delta_{ij}\tau(\mathbf{r}_q) \left\{ \frac{2}{3}(1 + jk_0u_q) \exp(-jk_0u_q) - 1 \right\}, \quad u_q = d_q \left(\frac{3}{4\pi} \right)^{1/3}
 \end{aligned}$$

- Approximate the probe current \mathbf{J}_a by the linear current at the central axis, apply the step functions as the expansions
- Apply the point-matching method at the probe surface
- δ gap excitation

CP excitation condition by ECM

$$Q_0 = \frac{k^2}{\gamma_{12}}$$

$$f_c = f_0 \left(1 + \frac{\gamma_{11}}{2k^2} \right)$$

$$\gamma_{11} = q_{11} - k^2 p_{11} = \gamma_{22}$$

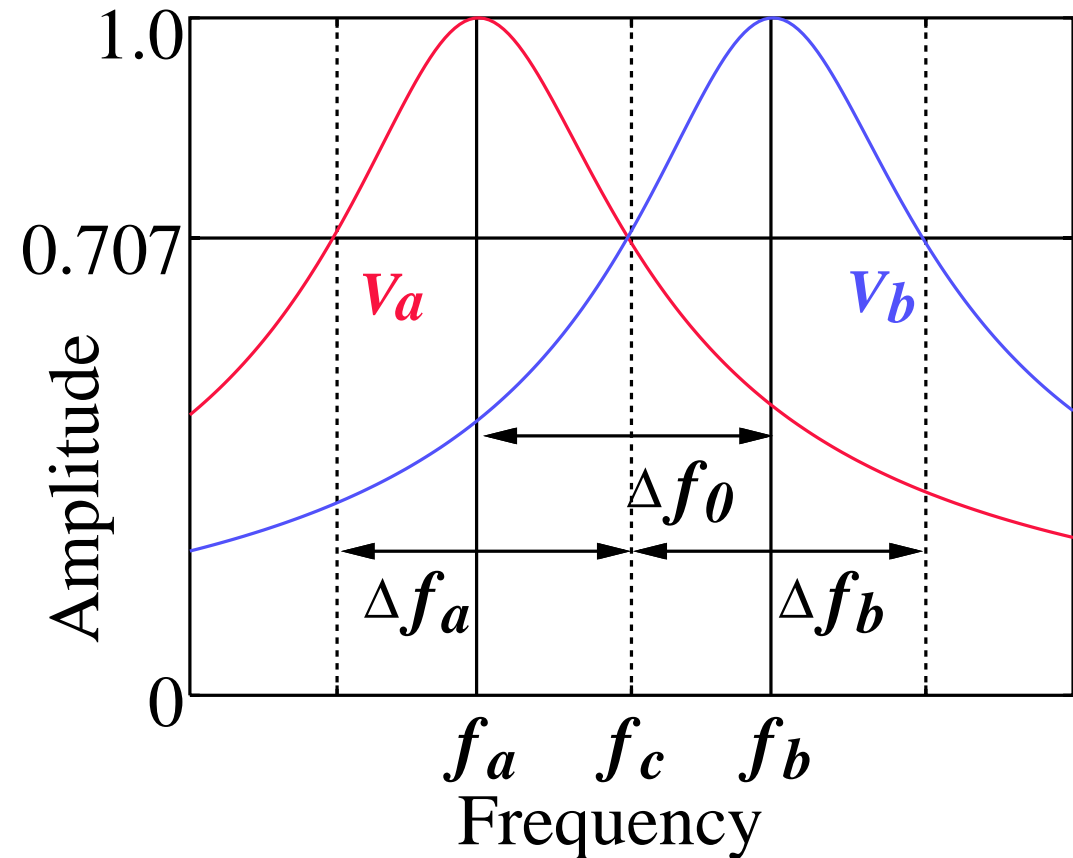
$$\gamma_{12} = q_{12} - k^2 p_{12}$$

$$p_{11} = \iiint_{\Delta V} \psi_a^2 dV = p_{22}$$

$$p_{12} = \iiint_{\Delta V} \psi_a \psi_b dV$$

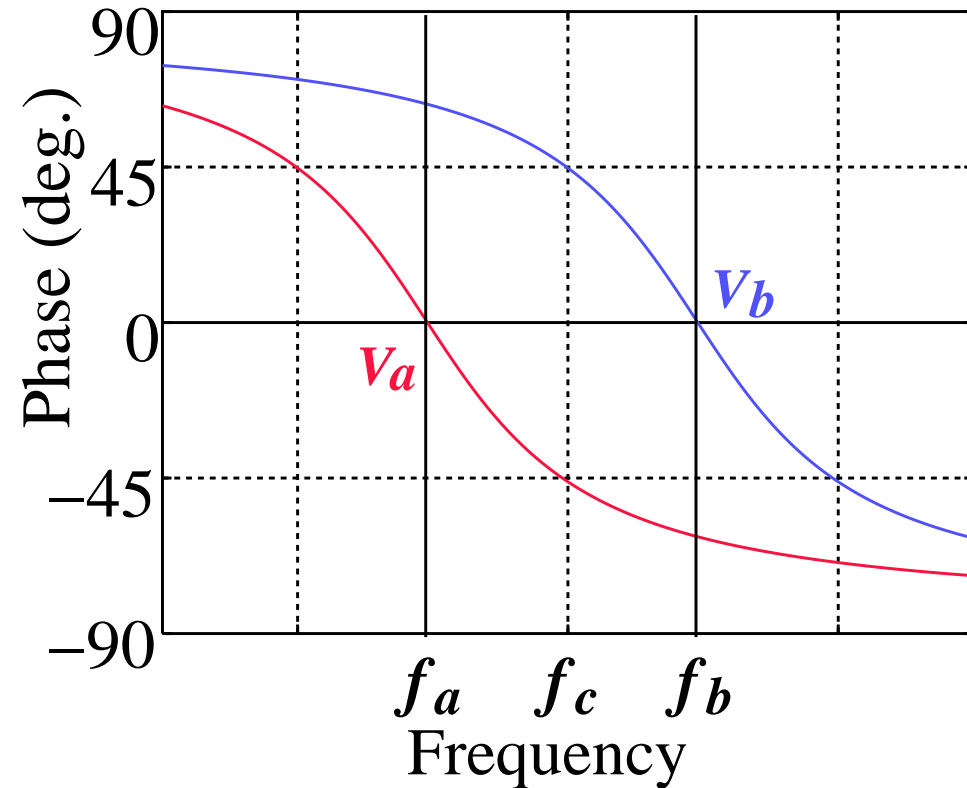
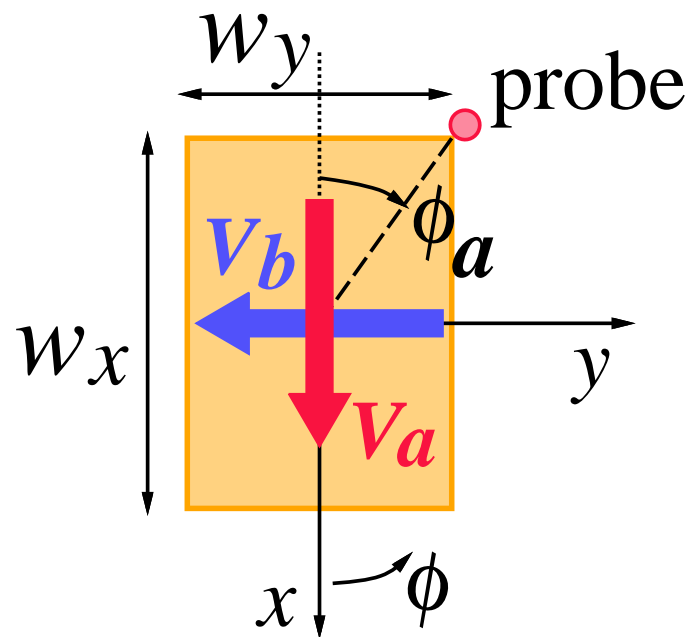
$$q_{11} = \iiint_{\Delta V} |\nabla \psi_a|^2 dV = q_{22}$$

$$q_{12} = \iiint_{\Delta V} \nabla \psi_a \cdot \nabla \psi_b dV$$



ψ_a : $\text{TE}_{1\delta 1}^y$ mode engen function when degenerate
 k : Eigenvalue when degenerate

Control of the right- and left circular polarized wave



$$w_x > w_y$$

- $0 < \phi_a < 90^\circ$, $180 < \phi_a < 270^\circ$

$$\mathbf{E} = E_0(\hat{x}V_a - \hat{y}jV_b)e^{-jk_0z} \quad \Rightarrow \text{right elliptic polarized wave}$$
- $90 < \phi_a < 180^\circ$, $270 < \phi_a < 360^\circ$

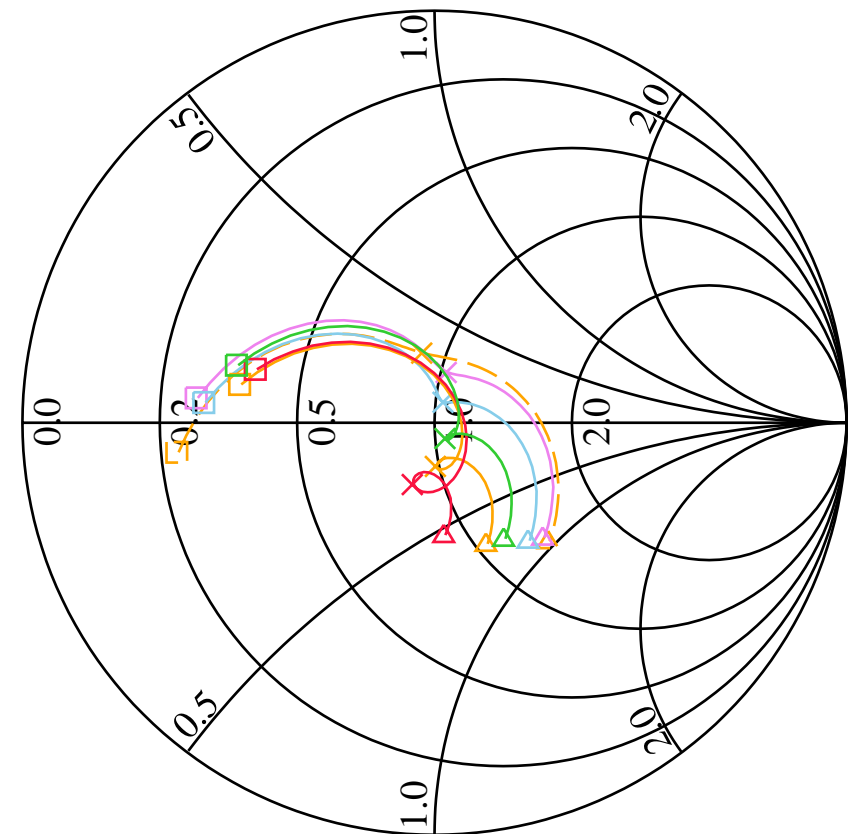
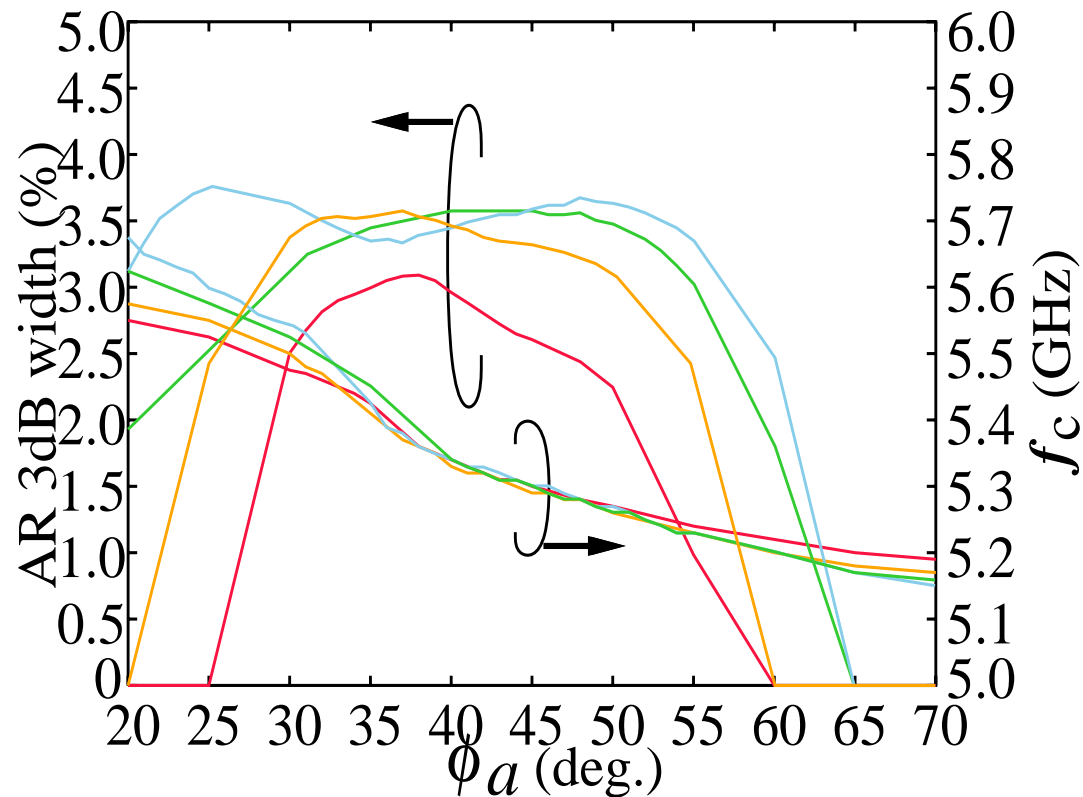
$$\mathbf{E} = E_0(\hat{x}V_a + \hat{y}jV_b)e^{-jk_0z} \quad \Rightarrow \text{left elliptic polarized wave}$$

ϕ_a vs. CP-DRA characteristics (2)

$\Delta w / w = 0.130$ — (red) $\Delta w / w = 0.160$ — (green)
 $\Delta w / w = 0.142$ — (orange) $\Delta w / w = 0.180$ — (blue)

ϕ_a (deg.) 40 — (red) 55 — (light blue)
 45 — (orange) 60 — (purple)
 48 — (green) 65 — (dashed orange)

□ 4.7 GHz Δ 5.7 GHz
 × minimum point of AR



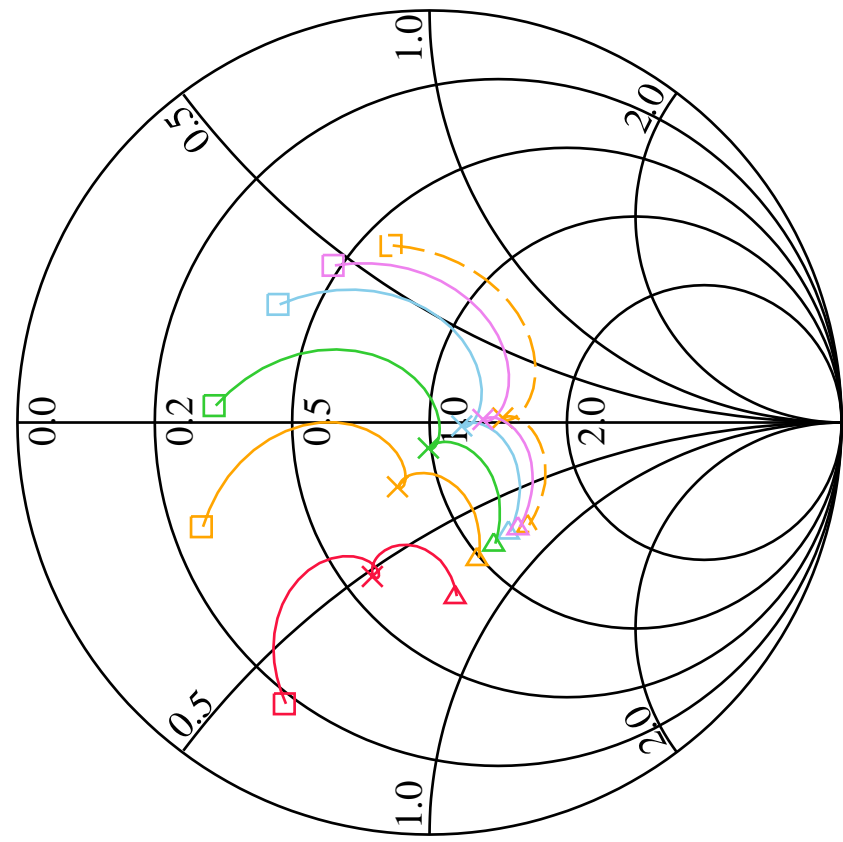
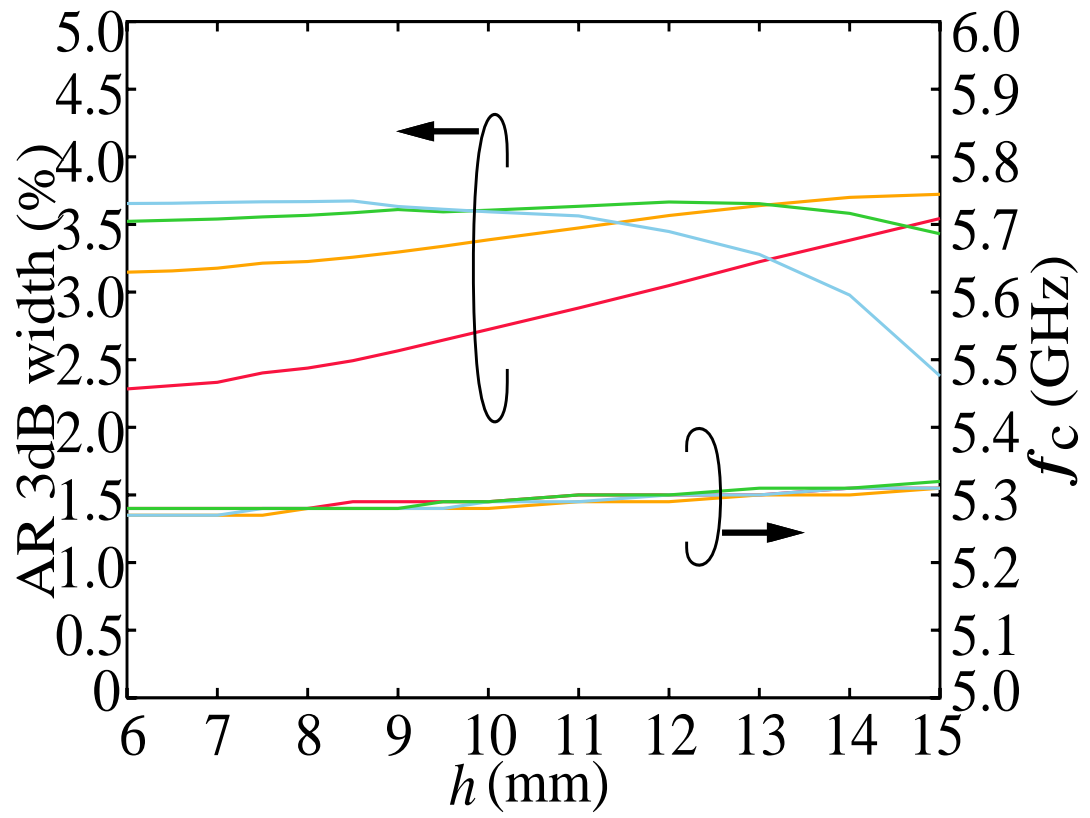
$\epsilon_r = 12.6$, $\tan \delta = 2.22 \times 10^{-5}$, $w_z = 8.0\text{mm}$, $w = 11.5\text{mm}$,
 $h = 8.0\text{mm}$, $d = 0.6\text{mm}$, $a = 0.5\text{mm}$

***h* vs. CP-DRA characteristics (2)**

$\Delta w / w = 0.130$ — (red) $\Delta w / w = 0.160$ — (green)
 $\Delta w / w = 0.142$ — (orange) $\Delta w / w = 0.180$ — (blue)

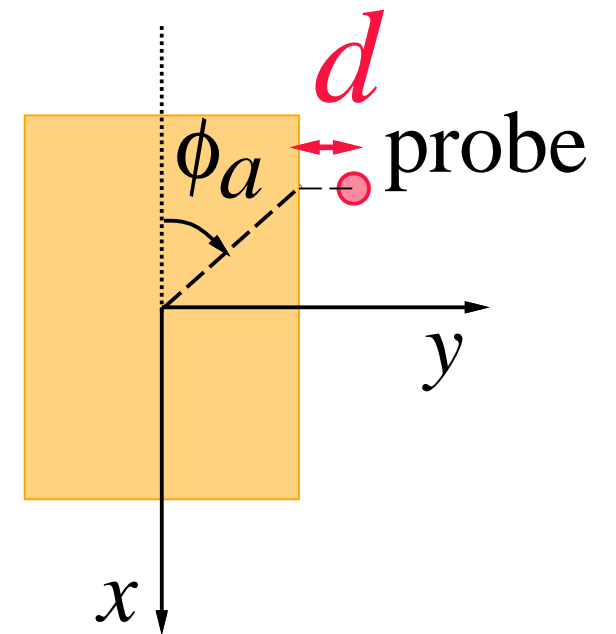
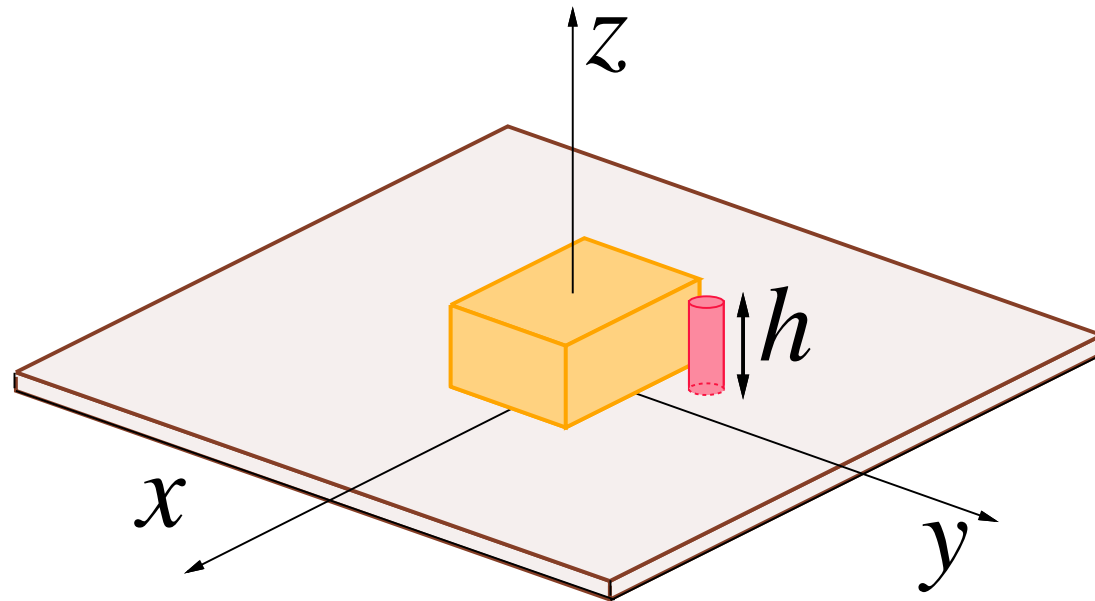
h (mm) 6 — (red) 9 — (light blue)
 7 — (orange) 10 — (purple)
 8 — (green) 11 — (dashed orange)

\square 4.7 GHz Δ 5.7 GHz
 \times minimum point of AR



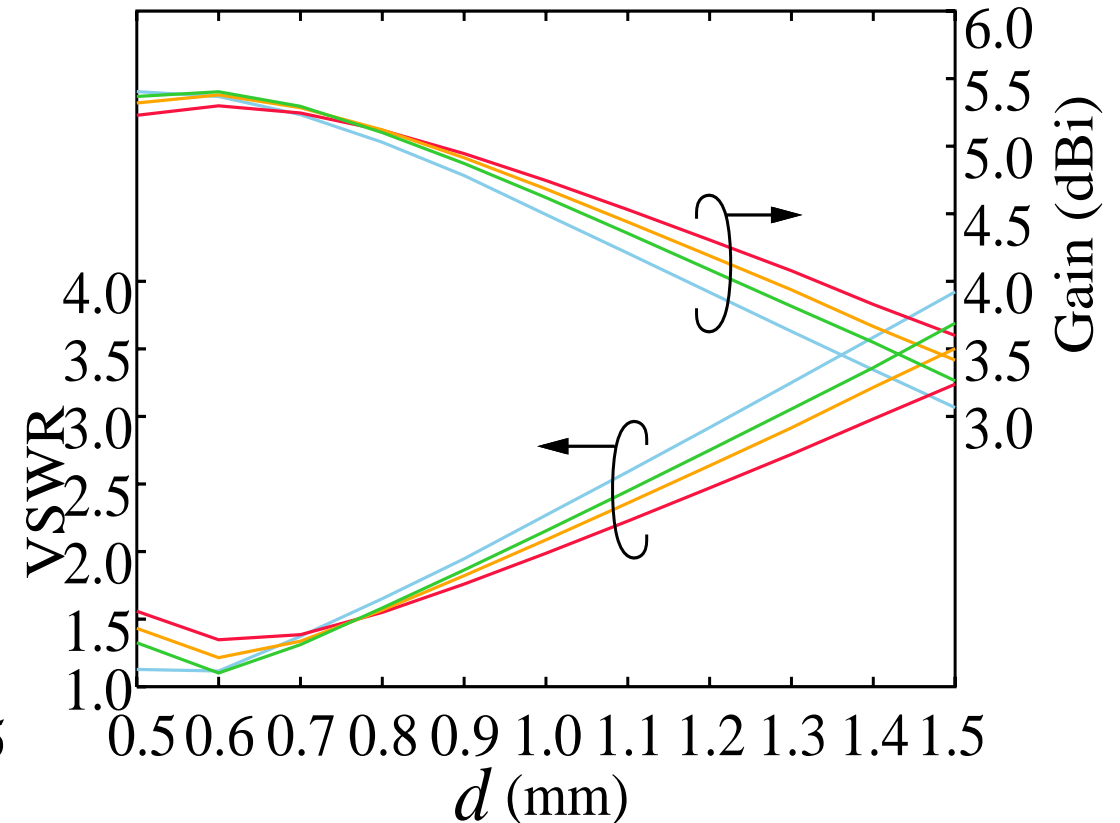
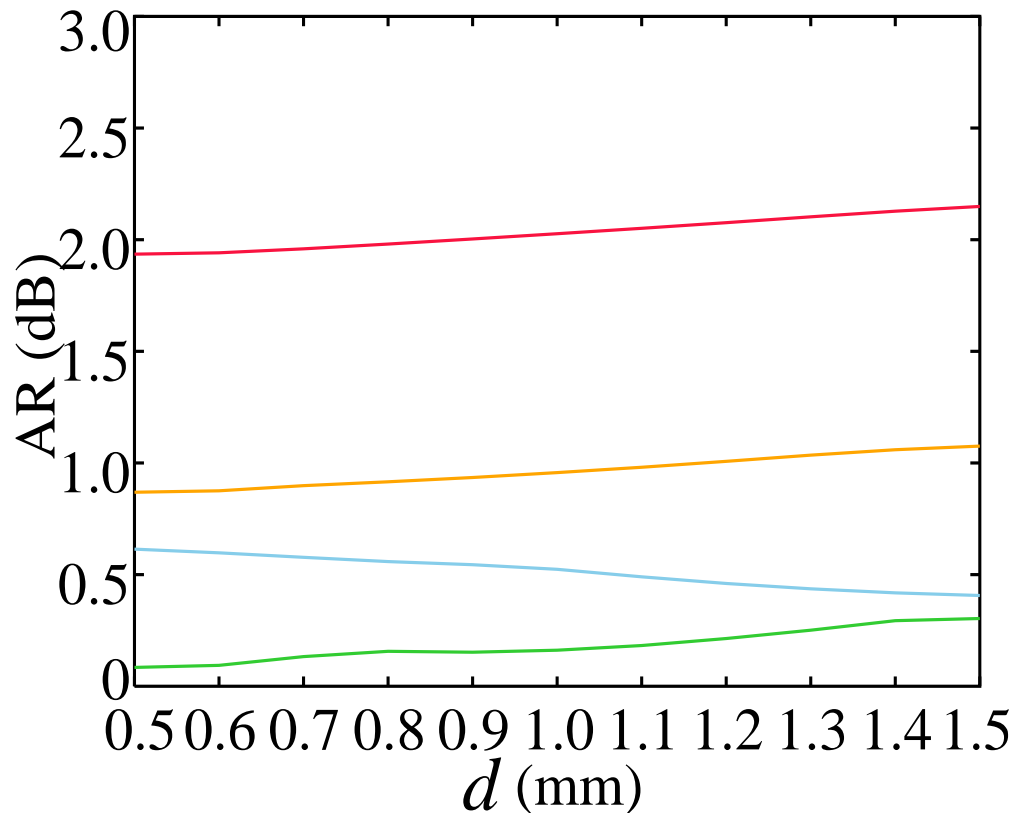
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Single probe-fed cuboidal CP-DRA



d vs. CP-DRA characteristics

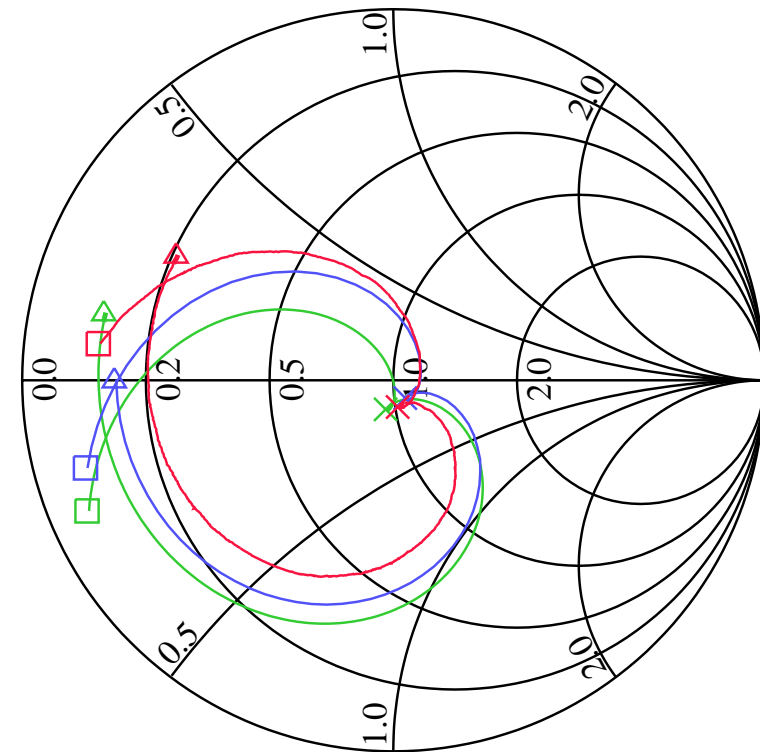
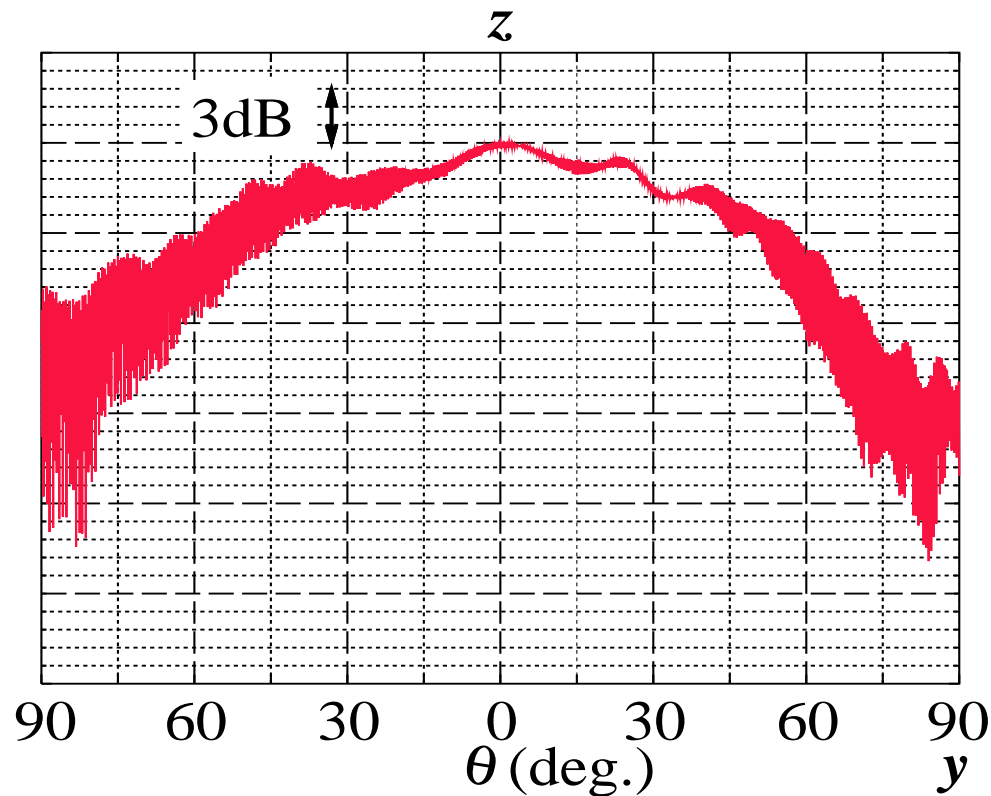
$\Delta w / w = 0.130$ — (red) $\Delta w / w = 0.160$ — (green)
 $\Delta w / w = 0.142$ — (orange) $\Delta w / w = 0.180$ — (blue)



$\epsilon_r = 12.6$, $\tan \delta = 2.22 \times 10^{-5}$, $w_z = 8.0\text{mm}$, $w = 11.5\text{mm}$,
 $\phi_a = 48^\circ$, $h = 8.0\text{mm}$, $a = 0.5\text{mm}$

Comparison with the measurement

measured ———
 M-PCMM ———
 HFSS ———
 □ 4.5 GHz △ 6.5 GHz
 × minimum point of AR

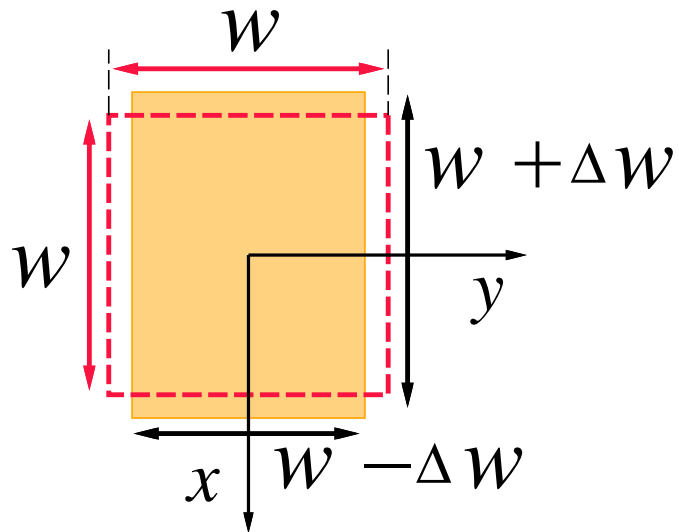


$$\begin{aligned}
 \epsilon_r = 12.6, \quad \tan \delta = 2.22 \times 10^{-5}, \quad w_z = 8.0\text{mm}, \quad w = 11.5\text{mm}, \\
 \Delta w/w = 0.16, \quad \phi_a = 48^\circ, \quad h = 8.0\text{mm}, \quad d = 0.6\text{mm}, \quad a = 0.5\text{mm}
 \end{aligned}$$

Parameters obtained at the second step of the design

$$w_x = w + \Delta w$$

$$w_y = w - \Delta w$$



Operation frequency

5.15 ~ 5.25GHz

Radio LAN

IEEE802.11a Standard

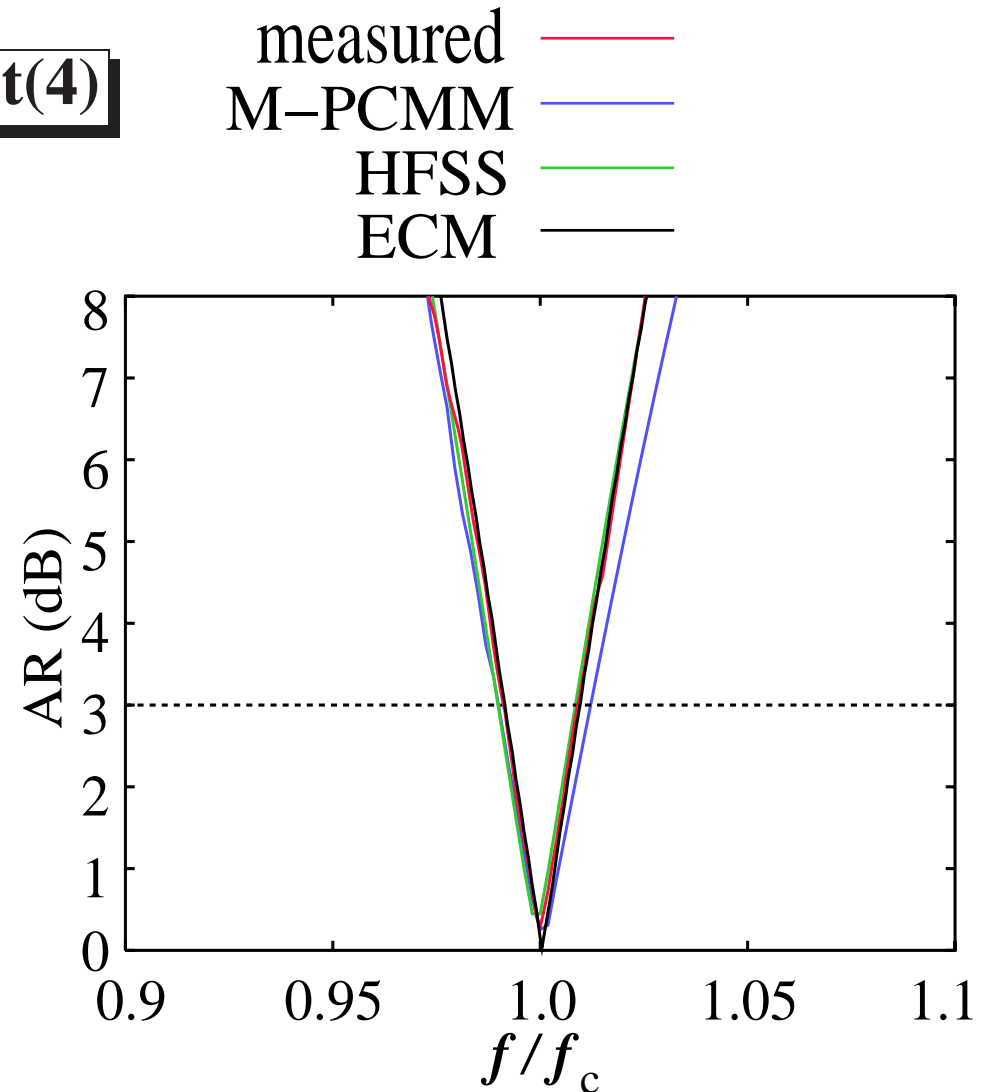
$$\varepsilon_r = 21.0, w_z = 7.0\text{mm}, w = 8.55\text{mm}$$

f_c (GHz)	$\Delta w/w$	w_x (mm)	w_y (mm)	w_z (mm)
5.160	0.0675	9.13	7.97	7.0
5.161	0.080	9.23	7.87	7.0
5.163	0.10	9.41	7.70	7.0
5.165	0.12	9.58	7.52	7.0

$$\phi_a = 60^\circ, h = 6.5\text{mm}, d = 0.6\text{mm}, a = 0.5\text{mm}$$

Comparison with the measurement(4)

	f_c (GHz)	AR (dB)	AR width(%)
Measured	5.30	0.390	1.80
M-PCMM	5.34	0.255	2.17
HFSS	5.23	0.448	1.87
ECM	5.16	0	1.81

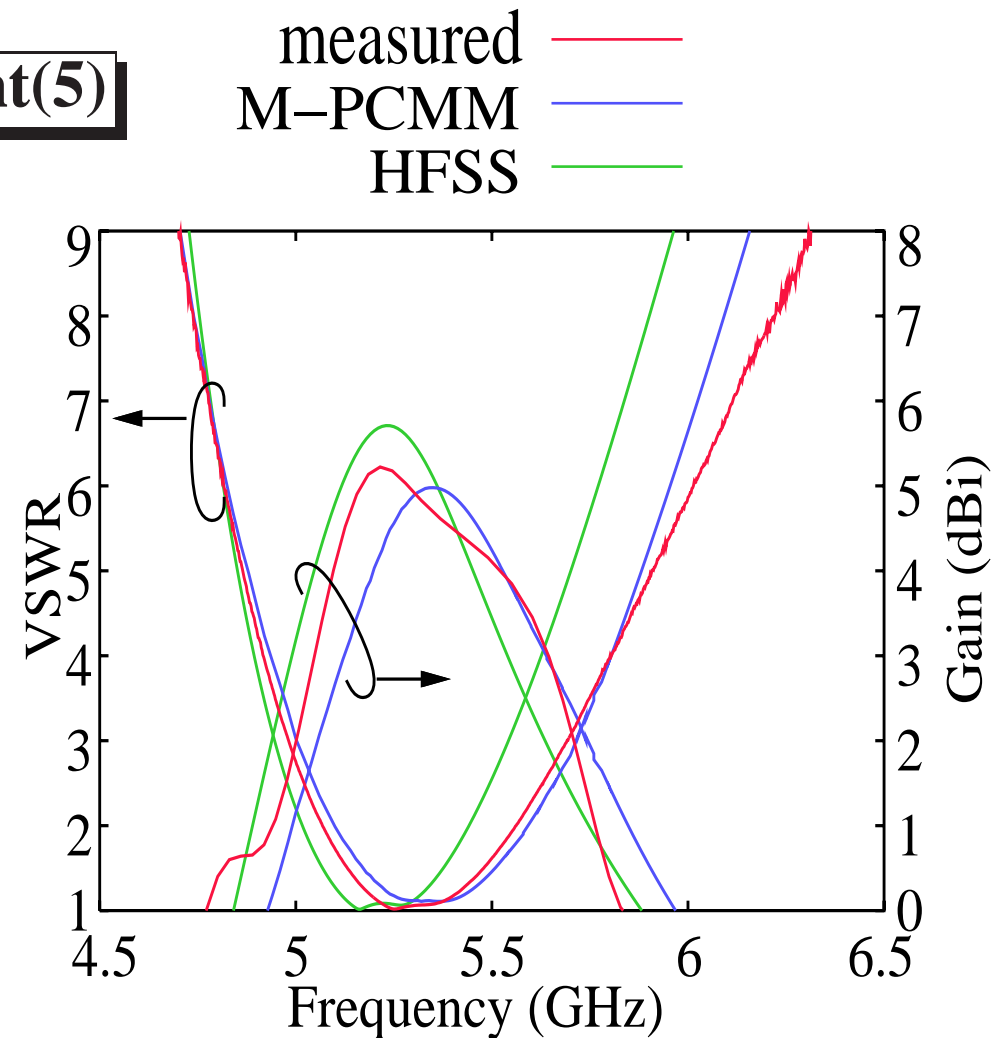


$$\varepsilon_r = 21.0, \tan \delta = 7.22 \times 10^{-5}, w_z = 7.0\text{mm}, w = 8.55\text{mm},$$

$$\Delta w/w = 0.10, \phi_a = 60^\circ, h = 6.5\text{mm}, d = 0.6\text{mm}, a = 0.5\text{mm}$$

Comparison with the measurement(5)

	f_c (GHz)	VSWR	VSWR width(%)
Measured	5.30	1.06	9.29
M-PCMM	5.34	1.11	9.23
HFSS	5.23	1.09	8.12
ECM	5.16	—	—

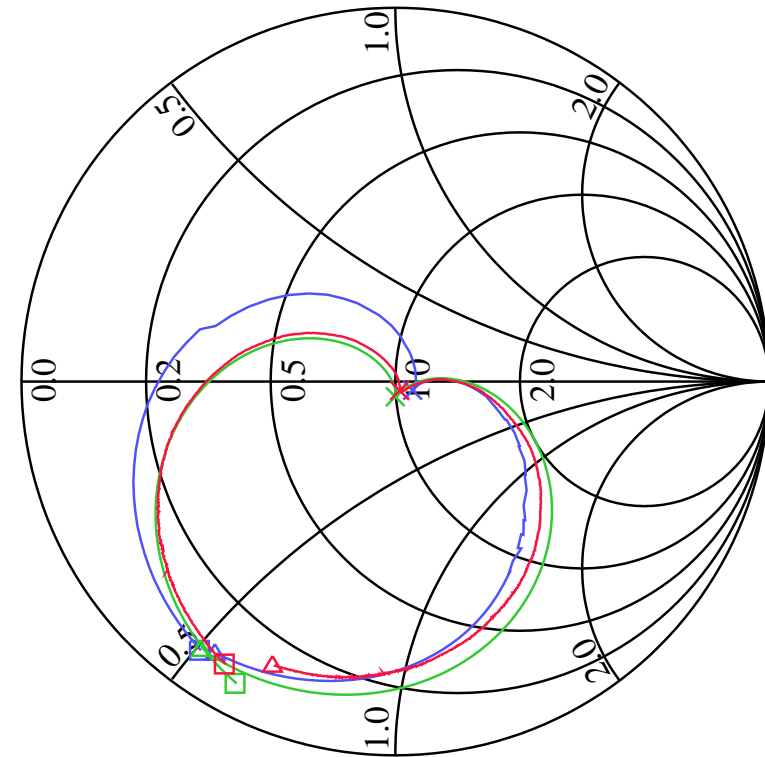
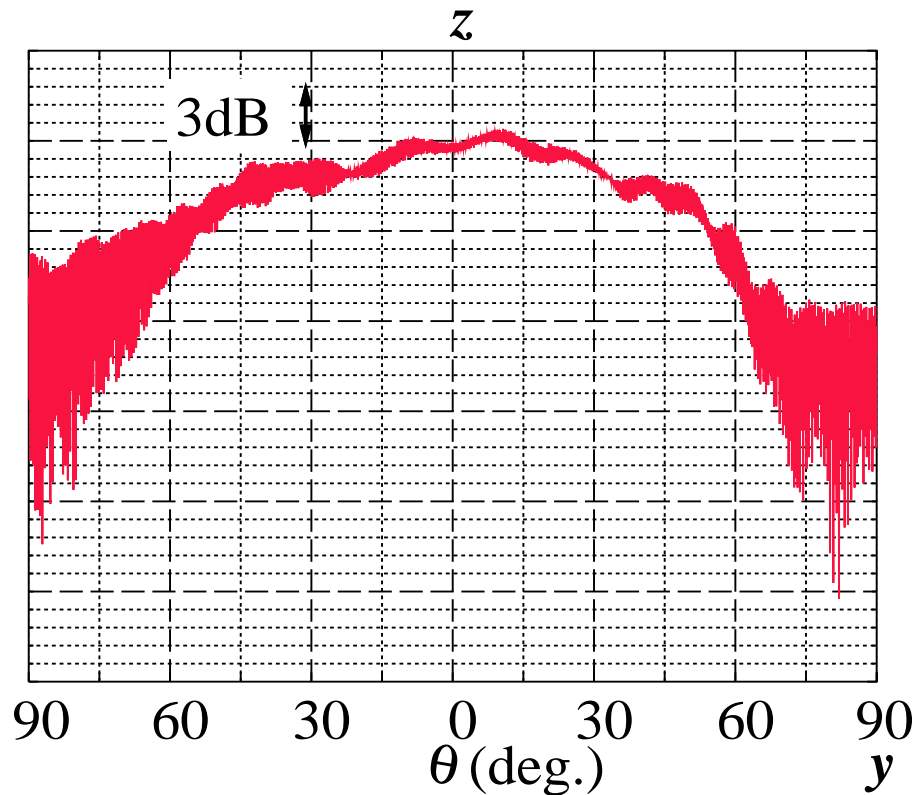


$$\varepsilon_r = 21.0, \tan \delta = 7.22 \times 10^{-5}, w_z = 7.0\text{mm}, w = 8.55\text{mm},$$

$$\Delta w/w = 0.10, \phi_a = 60^\circ, h = 6.5\text{mm}, d = 0.6\text{mm}, a = 0.5\text{mm}$$

Comparison with the measurement(6)

measured ——— (red line)
 M-PCMM ——— (blue line)
 HFSS ——— (green line)
 □ 4.5 GHz △ 6.5 GHz
 × minimum point of AR



$$\begin{aligned}
 \epsilon_r = 21.0, \quad \tan \delta = 7.22 \times 10^{-5}, \quad w_z = 7.0\text{mm}, \quad w = 8.55\text{mm}, \\
 \Delta w/w = 0.10, \quad \phi_a = 60^\circ, \quad h = 6.5\text{mm}, \quad d = 0.6\text{mm}, \quad a = 0.5\text{mm}
 \end{aligned}$$